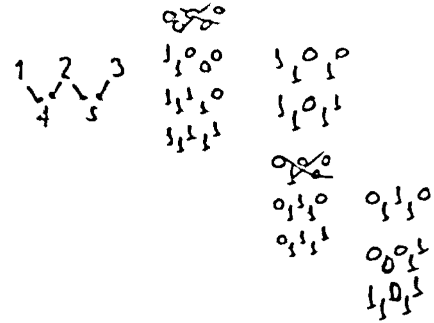
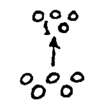
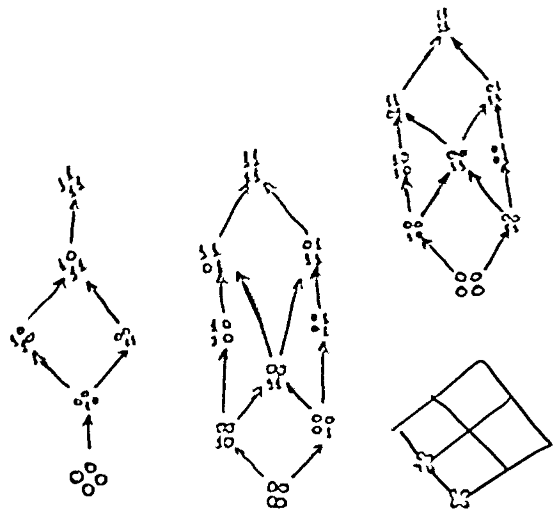
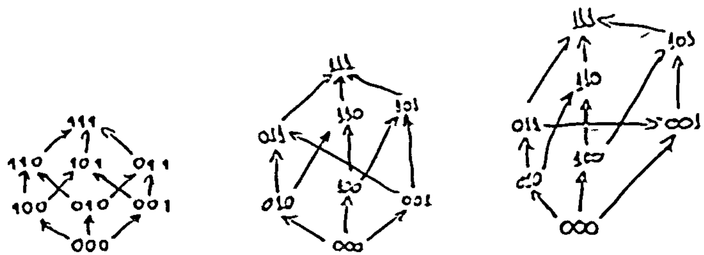
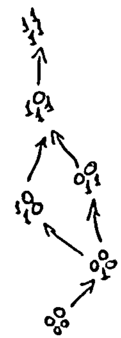


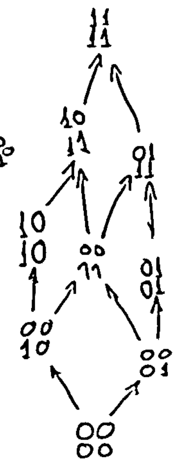








2·3



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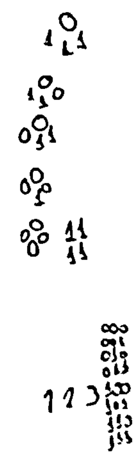
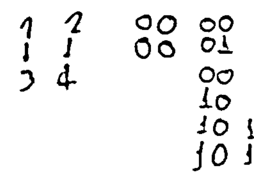
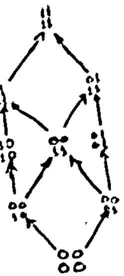
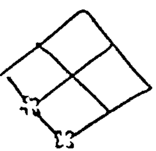
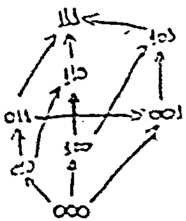


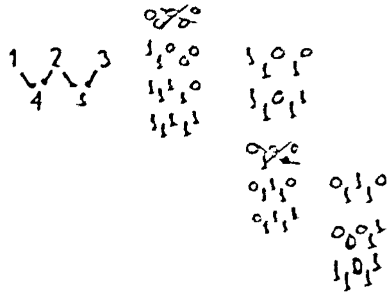
$$\underbrace{\underbrace{3\cdot 4}_{12} + \underbrace{5\cdot(\underbrace{6+\ldots}_{13})}_{65}}_{77}$$

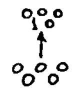


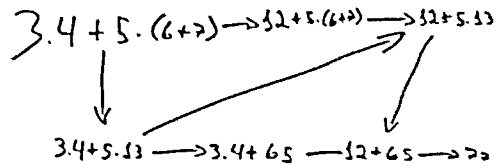


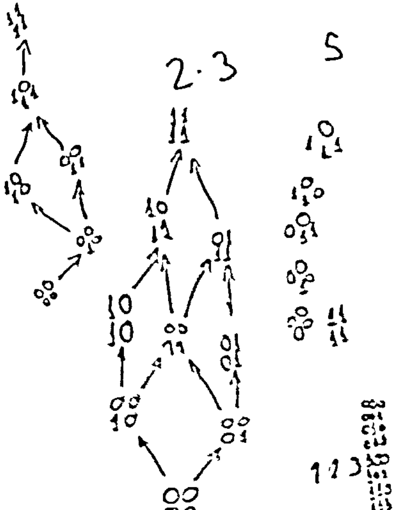




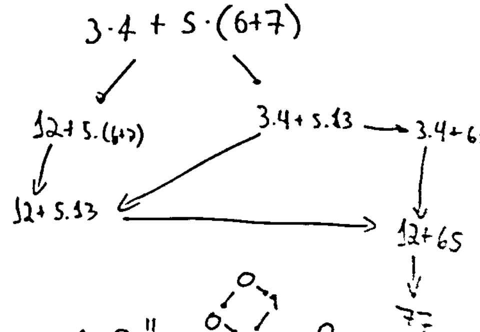


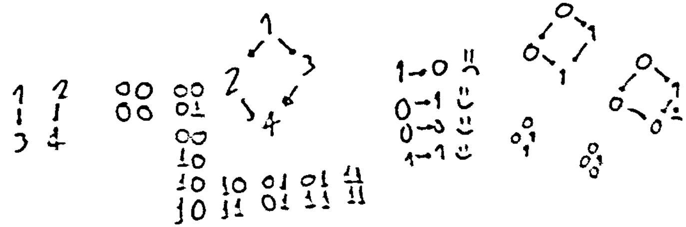


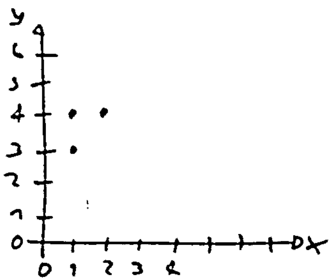




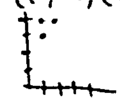
$$\underbrace{\underbrace{3\cdot 4}_{12} + \underbrace{5\cdot\underbrace{(6+7)}_{13}}_{65}}_{77}$$

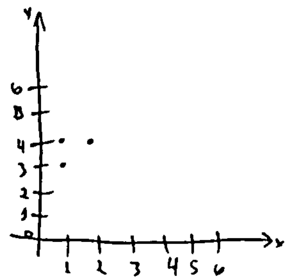




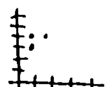


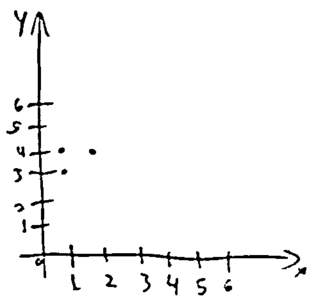
$A = \{(1,4),(2,4),(1,3)\}$

$=$ 

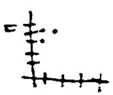


$B = \{(1,3),(1,4),(2,4)\}$

$=$ 

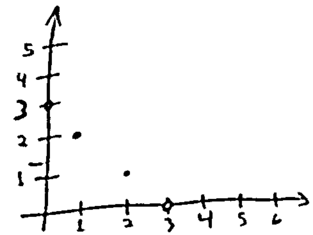


$C = \{(1,3),(1,4),(2,4),(2,4)\}$

$=$ 

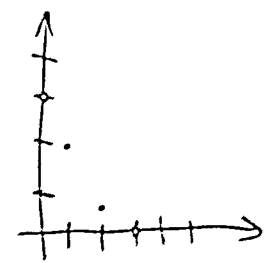
$K = \{x : \{0,1,2,3\};\ (x, 3-x)\} = \{(0,3),(1,2),(2,1),(3,0)\}$

x	$(x, 3-x)$
0	$(0,3)$
1	$(1,2)$
2	$(2,1)$
3	$(3,0)$



$m = \{y : \{0,1,2,3\};\ (3-y, y)\} = \{(3,0),(2,1),(1,2),(0,3)\}$

y	$(3-y, y)$
0	$(3,0)$
1	$(1,1)$
2	$(1,2)$
3	$(0,3)$

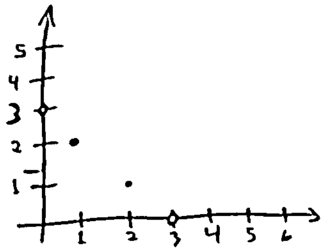


$g(2) -$

$g(2+3$

$(2+3)\cdot(2$

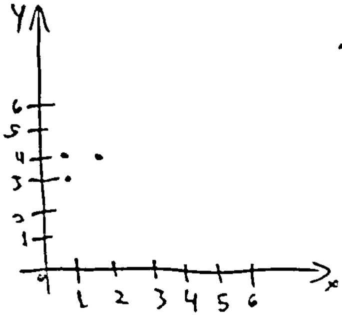
$5\cdot(2+3)+4$



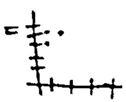
$m \cdot \{y : \{0,1,2,3\};\ (3-y,y)\}$

$= \{(3,0),(2,1),(1,2),(0,3)\}$

y	$(3-y,y)$
0	$(3,0)$
1	$(2,1)$
2	$(1,2)$
3	$(0,3)$



$C = \{(1,3),(1,4),(2,4),(2,4)\}$

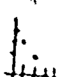


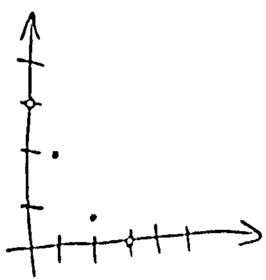
$\{0,1,2,3\};\ (x,3-x)\}$

$= \{(0,3),(1,2),(2,1),(3,0)\}$

$=$

$3-x$
$,3)$
$2)$
$1)$
$0)$





$g(a) \mapsto a \cdot a + 4$

$g(2+3)$ — $g(5)$

$(2+3)\cdot(2+3)+4$

$5\cdot(2+3)+4$ $\quad (2+3)\cdot 5+4$

$5 \cdot 5 + 4$

$25 + 4$

29

$h \mapsto \lambda a.\ a \cdot a + 4$

$h(2+3)$ — $h(5)$

$(\lambda a.\ a \cdot a + 4)(2+3)$ $\quad (\lambda a.\ a \cdot a + 4)(5)$

$(2+3)\cdot(2+3)+4$ $\quad 5 \cdot 5 + 4$

$25+4$

29

P	$\neg\neg P \to P$		$\neg\neg P \leftrightarrow P$	
00	00 11 00	00 11	00 11 00	00 11
01	01 00 11	01 01	01 00 11	01 01
11	11 00 11	11 11	11 00 11	11 11

$$\underbrace{\underbrace{3!}_{6} + \underbrace{4!}_{24}}_{30}$$

QUANDO $P=0$,

$$\underbrace{\neg\underbrace{\neg\underbrace{P}_{0}}_{1}}_{0} \to \underbrace{P}_{0}$$

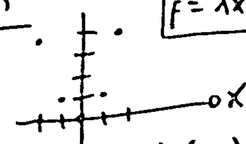
$$1$$

$$\boxed{\begin{aligned} f: \{-2,-1,0,1,2\} &\to \mathbb{N} \\ x &\mapsto x^2 \end{aligned}}$$

$$\boxed{f = \lambda x.x^2}$$

$$\boxed{f = \lambda x:\{-2,-1,0,1,2\}.x^2}$$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4



$F = \{(-2,4),(-1,1),(0,0),(1,1),(2,4)\}$

$f(3) = $ ERRO

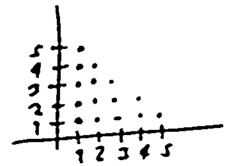
PORQUE $3 \notin \{-2,-1,0,1,2\}$

$F(-1) = 1$

$$f(1+1) \to f(2) \to \begin{Bmatrix}(-2,4),\\(-1,1),\\(0,0),\\(1,1),\\(2,4)\end{Bmatrix}(2) \to 4$$

$$(\lambda x:\{-2,-1,0,1,2\}.x^2)(1+1) \cdots\to (\lambda x.x^2)(1+1)$$

$$(1+1)^2 \to 2^2$$



$A = \{x:\{1,\ldots,5\},\ y:\{1,\ldots,5\},\ x+y \leq 6;\ (x,y)\}$

$\underbrace{3!}_{6} + \underbrace{4!}_{24}$ = 30

QUANDO $P=0$,

$\underbrace{\underbrace{\neg\underbrace{\neg \underbrace{P}_{0}}_{1}}_{0} \to \underbrace{P}_{0}}_{1}$

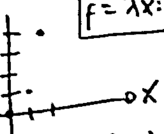
$\neg 0 = 1$
$\neg 1 = 0$

$0 \to 0 = 1$
$0 \to 1 = 1$
$1 \to 0 = 0$
$1 \to 1 = 1$

$\to \mathbb{N}$
$\to x^2$

$\boxed{f = \lambda x.x^2}$

$\boxed{f = \lambda x:\{-2,-1,0,1,2\}.x^2}$



$f(3)$ = ERRO porque $3 \notin \{-2,-1,0,1,2\}$

P	$\neg\neg P \to P$
0	$0\ 1\ 0$ 1 0
1	$1\ 0\ 1$ 1 1

4)),(-1,1),(0,0),
1),(2,4)}

= 1

$) \to \{(-2,4),(-1,1),(0,0),(1,1),(2,4)\}(2) \to 4$

$\to 2^2$

x	y	$x+y$	$x+y \leq 6$	(x,y)
1	1	2	1	(1,1)
	2	3	1	(1,2)
	3	4	1	(1,3)
	4	5	1	(1,4)
	5	6	1	(1,5)
2	1	3	1	(2,1)
	2	4	1	(2,2)
	3	5	1	(2,3)
	4	6	1	(2,4)
	5	7	0	
3	1	4	1	(3,1)
	2	5	1	(3,2)
	3	6	1	(3,3)
	4	7	0	
	5	8	0	
4	1	5	1	(4,1)
	2	6	1	(4,2)
	3	7	0	
	4	8	0	
	5	9	0	
5	1	6	1	(5,1)
	2	7	0	
	3	8	0	
	4	9	0	
	5	10	0	

$NV = 6 - n$

$n = x$

T = 

BM(T) = $\{(a,b):T,\ (c,d):T,\ b=d+1,\ a-c\in\{-1,0,1\},\ ((a,b),(c,d))\}$

DEF: BM(K) = $\{(a,b):K,\ (c,d):K,\ b=d+1,\ a-c\in\{-1,0,1\},\ ((a,b),(c,d))\}$

$$\begin{tikzcd}[column sep=small]
& & (2,2) \arrow[dl] \arrow[dr] & & \\
& (1,1) \arrow[dl] \arrow[d] & & (3,1) \arrow[d] \arrow[dr] & \\
(0,0) & (1,0) & & (3,0) & (4,0)
\end{tikzcd}$$

$$\begin{tikzcd}
& (1,3) \arrow[dl] \arrow[dr] & \\
(0,2) \arrow[dr] & & (2,2) \arrow[dl] \\
& (1,1) \arrow[d] & \\
& (1,0) &
\end{tikzcd}$$

$BM(T)=\{((2,2),(1,1)),\ ((2,2),(3,1)),\ ((1,1),(0,0)),\ ((1,1),(1,0)),\ \dots\}$

Seja $r_T: T\to\mathbb{N}$ esta função:

$$r_T=\left\{\begin{array}{c} ((2,2),1), \\ ((1,1),2),\quad ((3,1),3), \\ ((0,0),4), ((1,0),5),\quad ((3,0),6), ((4,0),7) \end{array}\right\}$$

$r_T((3,1))=3$

$r_T((3,0))=6$

NOTAÇÃO POSICIONAL:

$r_T=\begin{smallmatrix} &&1&& \\ &2&&3& \\ 4&5&&6&7 \end{smallmatrix}$

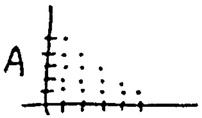
OBS: $\begin{smallmatrix} &1& \\ 2&&3 \\ &4& \\ &5& \end{smallmatrix}$:  $\to\mathbb{N}$

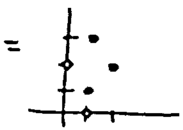
T = 

$r_K=\begin{smallmatrix} &1& \\ 2&&3 \\ &4& \\ &5& \end{smallmatrix}$

"r" é de "READING ORDER"

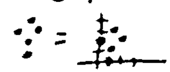
PODEMOS REPRESENTAR SUBCONJUNTOS FINITOS DE $\mathbb{N}^2$ COM DIAGRAMAS DE PONTINHOS...

A 

K = 

$=\{(0,2),(1,1),(1,3),(2,2),(1,0)\}$

CONVENÇÃO: QUANDO EU NÃO DESENHO OS EIXOS É PORQUE O CONJUNTO TOCA OS DOIS EIXOS.



UM GRAFO (DIRECIONADO) É UM PAR:

EX: $(\underbrace{\{1,2,3,4\}}_{A},\underbrace{\{(1,2),(1,3),(2,4),(3,4)\}}_{R\subseteq A\times A})$

$$\begin{tikzcd}
1 \arrow[r] \arrow[rr, bend left] & 2 \arrow[rrr, bend right] & 3 \arrow[r] & 4
\end{tikzcd}
\qquad
\begin{tikzcd}
1 \arrow[r] \arrow[d] & 2 \arrow[d] \\
3 \arrow[r] & 4
\end{tikzcd}$$

SEJAM

B = {1, 2},

C = {3, 4, 5

ENTÃO B×

{b:B, c:C

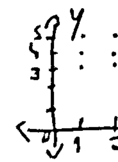
EXERCÍCIO:

CALCULE E

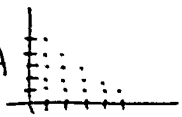
GRAFICAME

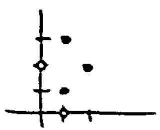
{ (1, 3),

(2, 3),



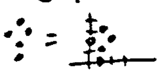
Podemos representar subconjuntos finitos de $\mathbb{N}^2$ com diagramas de pontinhos...

A 

K = 

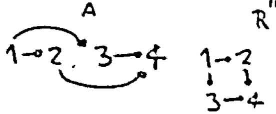
$= \{(0,2),(1,1),(1,3),(2,2),(1,0)\}$

Convenção: quando eu não desenho os eixos é porque o conjunto toca os dois eixos.



Um grafo (direcionado) é um par:

ex: $(\underbrace{\{1,2,3,4\}}_{A}, \underbrace{\{(1,2),(1,3),(2,4),(3,4)\}}_{R \subseteq A\times A})$



→ N

ING ORDER"

Sejam

$B=\{1,2\}$,

$C=\{3,4,5\}$.

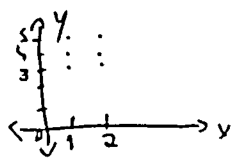
Então $B\times C =$

$\{b:B,\ c:C;\ (b,c)\}$.

Exercício:

Calcule e represente graficamente $B\times C$.

$\{(1,3),(1,4),(1,5),$
$(2,3),(2,4),(2,5)\}$



A =  $\subseteq \mathbb{N}^2 = \mathbb{N}\times\mathbb{N}$

$A = \{x:\{1,\dots,5\},\ y:\{1,\dots,5\},\ x+y\le 6;\ (x,y)\}$

x	y	$x+y$	$x+y\le 6$	(x,y)
1	1	2	1	(1,1)
	2	3	1	(1,2)
	3	4	1	(1,3)
	4	5	1	(1,4)
	5	6	1	(1,5)
2	1	3	1	(2,1)
	2	4	1	(2,2)
	3	5	1	(2,3)
	4	6	1	(2,4)
	5	7	0	
3	1	4	1	(3,1)
	2	5	1	(3,2)
	3	6	1	(3,3)
	4	7	0	
	5	8	0	
4	1	5	1	(4,1)
	2	6	1	(4,2)
	3	7	0	
	4	8	0	
	5	9	0	
5	1	6	1	(5,1)
	2	7	0	
	3	8	0	
	4	9	0	
	5	10	0	

$T =$ 

$BM(T) =$
$\{(a,b):T,$
$(c,d):T,$
$b = d+1,$
$a-c \in \{-1,0,1\},$
$((a,b),(c,d))\}$

DEF:
$BM(K) =$
$\{(a,b):K,$
$(c,d):K,$
$b = d+1,$
$a-c \in \{-1,0,1\},$
$((a,b),(c,d))\}$

$$\begin{tikzcd}
 & & (2,2) \arrow[dl] \arrow[dr] & & \\
 & (1,1) \arrow[dl] \arrow[d] & & (3,1) \arrow[d] \arrow[dr] & \\
(0,0) & (1,0) & & (3,0) & (4,0)
\end{tikzcd}$$

$$\begin{tikzcd}
 & (1,3) \arrow[dl] \arrow[dr] & \\
(0,2) \arrow[dr] & & (2,2) \arrow[dl] \\
 & (1,1) \arrow[d] & \\
 & (1,0) &
\end{tikzcd}$$

$BM(T) = \{((2,2),(1,1)), ((2,2),(3,1)),$
$((1,1),(0,0)), ((1,1),(1,0)),$
$\ldots\}$

Seja $r_T : T \to \mathbb{N}$
esta função:

$$r_T = \left\{ \begin{array}{ccc} & ((2,2),1), & \\ ((1,1),2), & & ((3,1),3), \\ ((0,0),4), ((1,0),5), & & ((3,0),6), ((4,0),7) \end{array} \right\}$$

$r_T((3,1)) = 3$
$r_T((3,0)) = 6$

NOTAÇÃO POSICIONAL:

$r_T = {}_{4\,5}{}^{2}{}^{1}{}^{3}{}_{6\,7}$

OBS: $\begin{smallmatrix} 2 & 1 & 3 \\ & 4 & \\ & 5 & \end{smallmatrix} :$  $\to \mathbb{N}$

$T =$ 

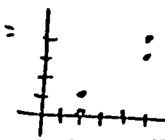
$r_K = \begin{smallmatrix} 2 & 1 & 3 \\ & 4 & \\ & 5 & \end{smallmatrix}$

"r" é de "READING ORDER"

DEF:
$f^{-1} = \{(a,b):f;\ (b$
Ex: Se $f = \{(0,1),(2,$
Então $f^{-1} = \{(1,0),($

$r_T^{-1}(3) = (3,1)$
$r_T^{-1}(4) = (0,0)$

OBS: Se $g = \{(0,2),(1,2),(3,$
Então $g^{-1} = \{(2,0),(2,1),$

$g^{-1} =$ 

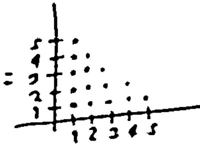
g^{-1} não é uma func

Como eu defino

Diga
Entã
DEF:

DEF: "

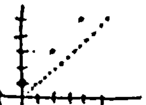
DEF: "

$A =$  $\subseteq \mathbb{N}^2 = \mathbb{N} \times \mathbb{N}$

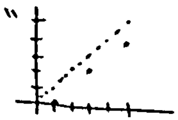
$A = \{x:\{1,\ldots,5\},\ y:\{1,\ldots,5\},\ x+y \le 6;\ (x,y)\}$

DEF:

$f^{-1} = \{(a,b):f;\ (b,a)\}$

EX: SE $f = \{(0,1),(2,3),(4,5)\} =$ 

ENTÃO $f^{-1} = \{(1,0),(3,2),(5,4)\}$

" 

SE $f:[0,\infty) \to [0,\infty)$
$x \mapsto x^2$

ENTÃO $f^{-1}:[0,\infty) \to [0,\infty)$
$x \mapsto \sqrt{x}$

$f =$ 

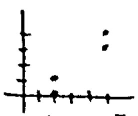
$f^{-1} =$ 

$r_T^{-1}(3) = (3,1)$

$r_T^{-1}(4) = (0,0)$

OBS: SE $g = \{(0,2),(1,2),(3,5),(4,5)\}$

ENTÃO $g^{-1} = \{(2,0),(2,1),(5,3),(5,4)\}$

$g^{-1} =$ 

g^{-1} NÃO É UMA FUNÇÃO!

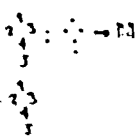
COMO EU DEFINO "f É UMA FUNÇÃO" $\to 1$
"g É UMA FUNÇÃO" $\to 0$

DIGAMOS QUE $h: A \to B$.
ENTÃO $h \subseteq A \times B$.

DEF: "h É UMA FUNÇÃO" $=$
$\forall a:A.\ \exists! b:B.\ (a,b) \in h$

DEF: "h É INJETIVA" $=$
$\forall a_1,a_2:A.\ h(a_1) = h(a_2) \to a_1 = a_2$

DEF: "h É SOBREJETIVA" $=$
$\forall b:B.\ \exists a:A.\ h(a) = b$.



DE "READING ORDER"

$\forall x:\{0,1,2\}.\ P(x) \to P(0) \,\&\, P(1) \,\&\, P(2)$

$\exists x:\{0,1,2\}.\ P(x) \to P(0) \lor P(1) \lor P(3)$

$\exists! x:\{0,1,2\}.\ P(x) \to (\exists x:\{0,1,2\}.\ P(x)) \,\&$
$(\forall a,b:\{0,1,2\}.\quad P(a) \,\&\, P(b) \to a=b)$

$(\exists! x:\{0,1,2\}.\ x>1) \to 1$

$(\exists! x:\{0,1,2\}.\ x>0) \xrightarrow{\text{no?}} 0$

$\downarrow$

$(\exists x:A.\ x>0) \,\&$
$(\forall a,b:A.\ a>0 \,\&\, b>0 \to a=b)$

$\downarrow$

$1 \,\&\quad 0$

a	b	$a>0$	$\&$	$b>0$	$\to$	$a=b$
0	0	0 0	·	1 1	1	
0	1	0 0	0	1	1	
0	2	0	0	1	1	
1	0	1	0	0	2	
1	1	1	1	1	1	$\frac{1}{0}$
1	2	2	2	2	0	
2	0					
2	1					
2	2					

SEJA $A = \{0,1,2\}$

$\underbrace{\underbrace{a>0}_{0} \,\&\, \underbrace{b>0}_{1}}_{0}$

$0 \to 1 \qquad a$

$\underbrace{\underbrace{\underbrace{a>0}_{1} \,\&\, \underbrace{b>0}_{1}}_{1} \to \underbrace{a=b}_{0}}_{0}$

$\quad a=1 \quad b=2$

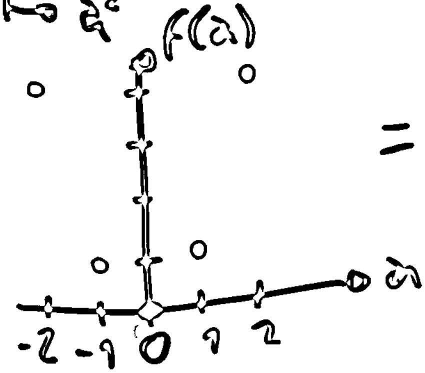
$f(a) = a^2$

Sezam: $A = \{-2, -1, 0, 1, 2\}$

$f: A \to \mathbb{N}$

$a \mapsto a^2$

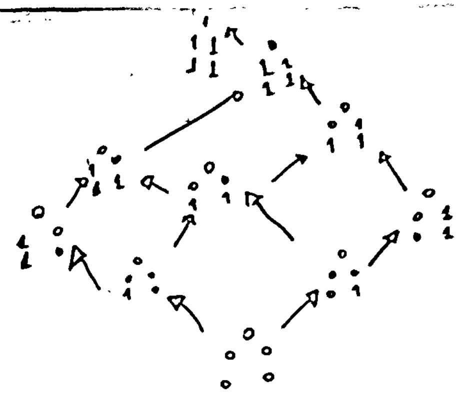
a	$f(a)$
-2	4
-1	1
0	0
1	1
2	4



$= \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

$(\lambda a : A . a^2) =$

$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$



Se $P=0$, $\underbrace{\underbrace{\neg\underbrace{\neg\underbrace{P}_{0}}_{1}}_{0} \to \underbrace{P}_{0}}_{1}$

Se $P=1$, $\underbrace{\underbrace{\neg\underbrace{\neg\underbrace{P}_{1}}_{0}}_{1} \to \underbrace{P}_{1}}_{1}$

$0 \to 0 \to 0 \to 1 \to 1 \to 1$

$1 \to 0$!!

$\neg\neg P \to P$

$0 \quad 1 \quad 0 \qquad 0$

1

P	$\neg\neg P \to P$
0	0 1 0 1 0
1	1 0 1 1 1

P	Q	$P \& Q \to P \vee Q$	$P \vee Q \to P \& Q$
0	0	0 0 0 1 0 0 0	0 0 0 1 0 0 0
0	1	0 0 1 1 0 1 1	0 1 1 0 0 0 1
1	0	1 0 0 1 1 1 0	1 1 0 0 1 0 0
1	1	1 1 1 1 1 1 1	1 1 1 1 1 1 1

Quando $a=6$ e $b=2$,

$$\underbrace{\underbrace{\underbrace{(\underbrace{a}_{6}+\underbrace{b}_{2})}_{8}\cdot\underbrace{(\underbrace{a}_{6}-\underbrace{b}_{2})}_{4}}_{32} = \underbrace{\underbrace{\underbrace{a}_{6}\cdot\underbrace{a}_{6}}_{36}-\underbrace{\underbrace{b}_{2}\cdot\underbrace{b}_{2}}_{4}}_{32}}_{\text{ok}}$$

$3 \in \mathbb{N}$? sim

$-3 \in \mathbb{N}$? não $-3 \in \mathbb{Z}$? sim

$\sqrt{2} \in \mathbb{N}$? não $\sqrt{2} \in \mathbb{Z}$? não $\sqrt{2} \in \mathbb{R}$? sim

$3+\sqrt{2} \in \mathbb{R}$?

$\sqrt{2} = 1.4142\ldots$

$3+\sqrt{2} = 4.4142\ldots$

$\sqrt{3} = 1.8\ldots$

$f(3) = 2+\sqrt{3}$

$\square : \square \to \square$

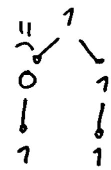
$\square \mapsto \square$

OPERAÇÕES
NOVAS:

$\underset{H}{\&}, \underset{H}{\vee}, \underset{H}{\to}, \underset{H}{\leftrightarrow}, \underset{H}{\neg}$

$\begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix} \underset{H}{\&} \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} = \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 0 \end{smallmatrix}$

$\begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix} \underset{H}{\to} \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} = \begin{smallmatrix} & 1 & \\ 0 & & 1 \\ 1 & & 1 \end{smallmatrix}$!!



$\begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix} \underset{H}{\to} \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} = \left(\begin{smallmatrix} & 1 & \\ 0 & & 1 \\ 1 & & 1 \end{smallmatrix}\right)^{INT}$

$= \begin{smallmatrix} & 0 & \\ 0 & & 1 \\ 1 & & 1 \end{smallmatrix}$

$\left(\begin{smallmatrix} & 1 & \\ 1 & & 1 \\ 1 & & 0 \end{smallmatrix}\right)^{INT} = \left(\begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix}\right)$

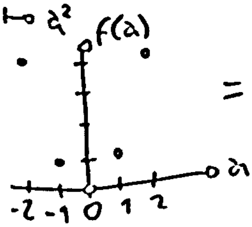
$\begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} \underset{H}{\to} \begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix} = \left(\begin{smallmatrix} & 1 & \\ 1 & & 1 \\ 1 & & 0 \end{smallmatrix}\right)^{INT}$

$= \begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix}$

$f(a) = a^2$

Sejam: $A = \{-2,-1,0,1,2\}$
$f: A \to \mathbb{N}$
$a \mapsto a^2$

a	$f(a)$
-2	4
-1	1
0	0
1	1
2	4



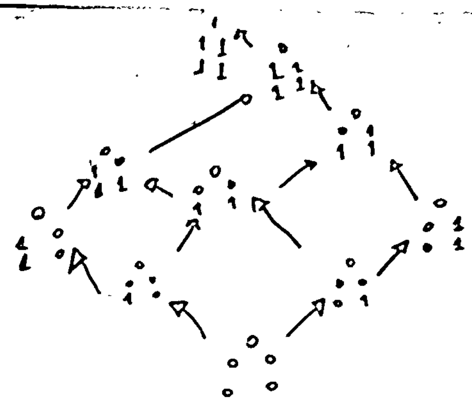
$= \{(-2,4), (-1,1), (0,0), (1,1), (2,4)\}$

$(\lambda a{:}A.\,a^2) =$
$\{(-2,4),(-1,1),(0,0),(1,1),(2,4)\}$

P

O

1



H = ⁖ ("HOUSE")

K = ⁘ ("KITE")

G = ⁙ ("GUILL")

$\begin{smallmatrix}0\\1\ 0\\1\ 0\end{smallmatrix} \mathbin{\&_H} \begin{smallmatrix}0\\0\ 1\\1\ 1\end{smallmatrix} = \begin{smallmatrix}0\\0\ 0\\1\ 0\end{smallmatrix}$ $\qquad$ $\begin{smallmatrix}0\\0\ 0\\1\ 1\end{smallmatrix} \mathbin{\&_H} \begin{smallmatrix}0\\0\ 1\\1\ 1\end{smallmatrix} = \begin{smallmatrix}0\\0\ 0\\1\ 1\end{smallmatrix}$

$\left(\begin{smallmatrix} & 1 & \\ 1 & & 1\\ 1 & & 0\end{smallmatrix}\right)^{INT}$

$\begin{smallmatrix}0\\1\ 0\\1\ 0\end{smallmatrix}$

Se $P=0$, $\underbrace{\underbrace{\neg\underbrace{\neg\underbrace{P}_{0}}_{1}}_{0} \multimap \underbrace{P}_{0}}_{1}$ $\qquad$ Se $P=1$, $\underbrace{\underbrace{\neg\underbrace{\neg\underbrace{P}_{1}}_{0}}_{1} \to \underbrace{P}_{1}}_{1}$

$0 \to 0 \to 0 \multimap 1 \to 1 \multimap 1$

$1 \to 0$!!

$\neg\neg P \multimap P$

$0\ {}^{1}\ {}^{0}\ \ 0$

1

P	¬¬P ⊸ P
0	0 1 0 0 1
1	1 0 1 1 1

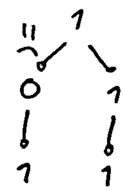
P	Q	P&Q → P∨Q	P∨Q → P&Q
0	0	000 1 000	000 1 000
0	1	001 1 011	111 0 101
1	0	100 1 110	110 0 100
1	1	111 1 111	111 1 111

OPERAÇÕES NOVAS:

$$\underset{H}{\&},\ \underset{H}{\vee},\ \underset{H}{\multimap},\ \underset{H}{\leftrightarrow},\ \underset{H}{\neg}$$

$$\begin{smallmatrix} & 0 \\ 1 & 0 \\ 1 & 0 \end{smallmatrix} \underset{H}{\&} \begin{smallmatrix} & 0 \\ 0 & 0 \\ 1 & 1 \end{smallmatrix} = \begin{smallmatrix} & 0 \\ 0 & 0 \\ 1 & 0 \end{smallmatrix}$$

$$\begin{smallmatrix} & 0 \\ 1 & 0 \\ 1 & 0 \end{smallmatrix} \underset{H}{\multimap} \begin{smallmatrix} & 0 \\ 0 & 0 \\ 1 & 1 \end{smallmatrix} = \begin{smallmatrix} & 1 \\ 0 & 1 \\ 1 & 1 \end{smallmatrix}$$



$$\begin{smallmatrix} & 0 \\ 1 & 0 \\ 1 & 0 \end{smallmatrix} \underset{H}{\multimap} \begin{smallmatrix} & 0 \\ 0 & 0 \\ 1 & 1 \end{smallmatrix} = \left(\begin{smallmatrix} & 1 \\ 0 & 1 \\ 1 & 1 \end{smallmatrix}\right)^{INT} = \begin{smallmatrix} & 0 \\ 0 & 1 \\ 1 & 1 \end{smallmatrix}$$

$0 \leq 0 = 1$

$0 \leq 1 = 1$

$1 \leq 0 = 0$

$1 \leq 1 = 1$

"$\leq$" É PARECIDA COM "$\multimap$"!

$$\begin{smallmatrix} & 0 \\ 1 & 0 \\ 1 & 0 \end{smallmatrix} \underset{H}{\leq} \begin{smallmatrix} & 0 \\ 0 & 0 \\ 1 & 1 \end{smallmatrix} = \begin{pmatrix} & (0\leq 0)\& \\ (1\leq 0)\& & (0\leq 0)\& \\ (1\leq 1)\& & (0\leq 1) \end{pmatrix} = \begin{pmatrix} & 1\& \\ 0\& & 1\& \\ 1\& & 1 \end{pmatrix} = 0$$

$$\underbrace{\underbrace{0\&1}_{0}\&\underbrace{0\&1}_{0}}_{0}$$

$$P = \begin{smallmatrix} & P_1 \\ P_2 & P_3 \\ P_4 & P_5 \end{smallmatrix}$$

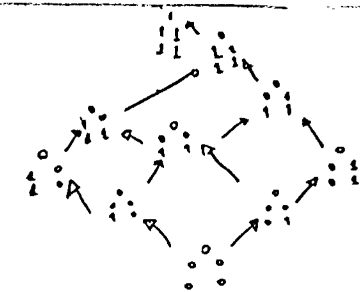
SE $P = \begin{smallmatrix} & 0 \\ 1 & 0 \\ 1 & 0 \end{smallmatrix}$ ENTÃO: $P_1 = 0$, $P_2 = 1$, $P_3 = 0$, $P_4 = 1$, $P_5 = 0$

$$P \underset{H}{\leq} Q$$

$H = \begin{smallmatrix} & \bullet & \\ \bullet & & \bullet \\ \bullet & & \bullet \end{smallmatrix}$ ("HOUSE")

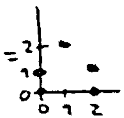
$K = \begin{smallmatrix} & \bullet & \\ \bullet & & \bullet \\ & \bullet & \\ & \bullet & \end{smallmatrix}$ ("KITE")

$G = \begin{smallmatrix} & \bullet & \bullet \\ \bullet & \bullet & \\ & \bullet & \bullet \end{smallmatrix}$ ("GUILL")



$\begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix} \;\&_H\; \begin{smallmatrix} & 0 & \\ 0 & & 1 \\ 1 & & 1 \end{smallmatrix} = \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 0 \end{smallmatrix}$ $\qquad$ $\begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} \;\&_H\; \begin{smallmatrix} & 0 & \\ 0 & & 1 \\ 1 & & 1 \end{smallmatrix} = \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix}$

$$\mathcal{O}(H) = \left\{ \begin{smallmatrix} & 1 & \\ 1 & & 1 \\ 1 & & 1 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 1 & & 1 \\ 1 & & 1 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 1 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 0 & & 1 \\ 1 & & 1 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 1 & & 0 \\ 1 & & 0 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 0 & & 1 \\ 0 & & 1 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 0 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 0 & & 1 \end{smallmatrix},\ \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 0 & & 0 \end{smallmatrix} \right\}$$

$H =$ 

$$= \left\{ \begin{matrix} & (1,2), & \\ (0,1), & & (2,1), \\ (0,0), & & (2,0) \end{matrix} \right\}$$

$f : H \to \mathbb{N}$, $(x,y) \mapsto x$

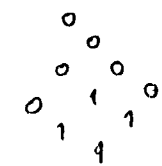
$g : H \to \mathbb{N}$, $(x,y) \mapsto y$

$h : H \to \mathbb{N}$, $(x,y) \mapsto x+y$

$$f = \left\{ \begin{matrix} & ((1,2),1), & \\ ((0,1),0), & & ((2,1),2), \\ ((0,0),0), & & ((2,0),2) \end{matrix} \right\} = \begin{smallmatrix} & 1 & \\ 0 & & 2 \\ 0 & & 2 \end{smallmatrix} = \lambda(x,y){:}H.x$$

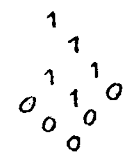
$$g = \left\{ \begin{matrix} & ((1,2),2) & \\ ((0,1),1), & & ((2,1),1), \\ ((0,0),0), & & ((2,0),0) \end{matrix} \right\} = \begin{smallmatrix} & 2 & \\ 1 & & 1 \\ 0 & & 0 \end{smallmatrix} = \lambda(x,y){:}H.y$$

$$h = \left\{ \begin{matrix} & ((1,2),3) & \\ ((0,1),1), & & ((2,1),3) \\ ((0,0),0) & & ((2,0),2) \end{matrix} \right\} = \begin{smallmatrix} & 3 & \\ 1 & & 3 \\ 0 & & 2 \end{smallmatrix} = \lambda(x,y){:}H.x+y$$

$\lambda P{:}\mathcal{O}(H).\ P \le_H \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} =$ 

$\begin{smallmatrix} & 1 & \\ 1 & & 1 \\ 1 & & 1 \end{smallmatrix} \le_H \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} = 0$

$\begin{smallmatrix} & 0 & \\ 1 & & 1 \\ 1 & & 1 \end{smallmatrix} \le_H \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} = 0$

$\lambda Q{:}\mathcal{O}(H).\ \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} \le_H Q =$ 

$\begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} \le_H \begin{smallmatrix} & 1 & \\ 1 & & 1 \\ 1 & & 1 \end{smallmatrix} = 1$

$\begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} \le_H \begin{smallmatrix} & 0 & \\ 1 & & 1 \\ 1 & & 1 \end{smallmatrix} = 1$

$\lambda P{:}\mathcal{O}(H).\ P \;\&_H\; \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} =$ 

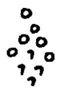
$\begin{smallmatrix} & 1 & \\ 1 & & 1 \\ 1 & & 1 \end{smallmatrix} \;\&_H\; \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix} = \begin{smallmatrix} & 0 & \\ 0 & & 0 \\ 1 & & 1 \end{smallmatrix}$

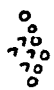
$P \text{ below } Q \;=\; P \leq_H Q$

$P \text{ leftof } Q \;=\; P_\ell \geq Q_\ell \;\&\; P_r \leq Q_r$

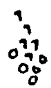
$P \text{ rightof } Q \;=\; P_\ell \leq Q_\ell \;\&\; P_r \geq Q_r$

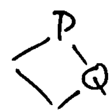
$P \text{ above } Q \;=\; P \geq_H Q$

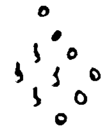
$\lambda P.\ P \text{ below } 11 =$ 

$\lambda P.\ P \text{ leftof } 11 =$ 

$\lambda P.\ P \text{ rightof } 11 =$ 

$\lambda P. P \text{ above } 11 =$ 



$\lambda P : H'.\ P \text{ leftof } 11 =$ 

$1 \geq 1 \;\&\; 1 \leq 1$

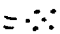
$1 \;\&\; 1$

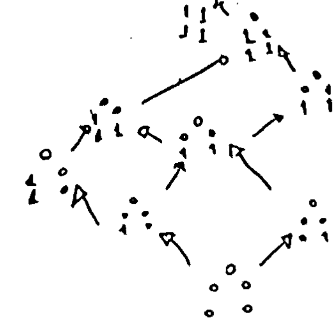
1

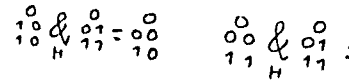
P	$P_\ell \geq 1 \;\&\; P_r \leq 1$
32	0
22	0
21	1
12	0
20	1
11	1
02	0
10	1
01	0
00	0

$H =$  ("HOUSE")

$K =$  ("KITE")

$G =$  ("GULL")





$P = \begin{matrix} & P_1 & \\ P_2 & & P_3 \\ P_4 & & P_5 \end{matrix}$

$P = P_\ell P_r$

$P = 32 \multimap P_\ell = 3$

$P_r = 2$

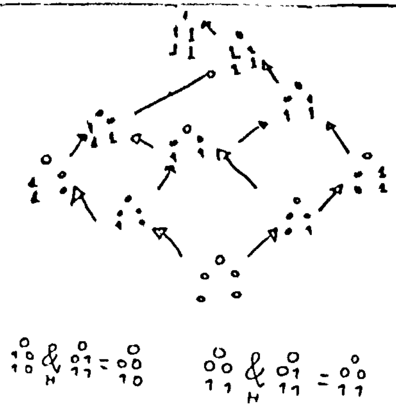
$P \vee_{H'} Q = \max(P_\ell, Q_\ell)\max(P_r, Q_r)$

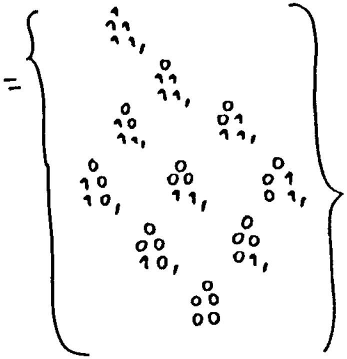
P	Q	$P \vee_{H'} Q$
20	12	22
32	22	32
01	32	32
20	01	21

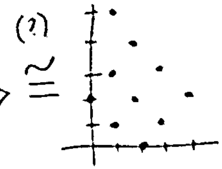
$P \,\&_{H'}\, Q = (\min(P_\ell, Q_\ell))(\min(P_r, Q_r))$

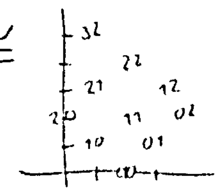
P	Q	$\min(P_\ell,Q_\ell)\min(P_r,Q_r)$	$P \,\&_{H'}\, Q$
20	12	1 2 1 0 0 2	10
32	22		22
01	32		01
20	01		00

H = ⁘ ("HOUSE")
K = ⁘ ("KITE")
G = ⁘ ("GULL")



$O(H) =$ 

$\cong$  $= \{(1,3), (2,4), (1,3), (3,3), (0,2), (2,2), (4,2), (1,1), (3,1), (2,0)\}$

$\cong$  $H' = \{32, 22, 21, 12, 20, 11, 02, 10, 01, 00\}$

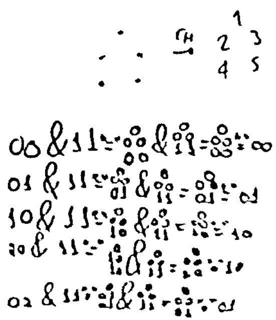
$P = \begin{matrix} & P_1 & \\ P_2 & & P_3 \\ P_4 & & P_5 \end{matrix}$

$P = P_\ell P_r$

$P = 32 \to P_\ell = 3$
$P_r = 2$

$P \underset{H'}{\&} Q = (\min(P_\ell, Q_\ell))(\min(P_r, Q_r))$

$\min(P_\ell,Q_\ell)$	$\min(P_r,Q_r)$	$P \underset{H'}{\&} Q$
1 2 1	0 0 2	10
		22
		01
		00



$\lambda P : O(H).\ P \underset{H}{\leq} \ldots$

$\lambda P : H'.\ P \underset{H'}{\leq} 12 = \ldots$

$\lambda P : H'.\ P \underset{H'}{\&} 11 = \ldots$

$32 \underset{H'}{\&} 11$ "=" $\ldots$

"=" 11

$22 \underset{H'}{\&} 11$ "=" $\ldots$

$11 \underset{H'}{\&} 11 = 11$

$22 \underset{H'}{\&} 11 = 11$

$21 \underset{H'}{\&} \ldots 11$

$12 \underset{H'}{\&} 11 = 11$

QUINDI $f = \lambda x.\, 4x+2$

$$f(10) \to (\lambda x.\, 4x+2)(10)$$
$$\downarrow$$
$$4 \cdot 10 + 2$$
$$\downarrow$$
$$40+2$$
$$\downarrow$$
$$42$$

$$2\cdot(3+4) + 5\cdot 6 \longrightarrow$$
$$\downarrow$$
$$2\cdot 7 + 5\cdot 6 \to 2$$
$$\downarrow$$
$$14 + 5\cdot 6$$
$$\downarrow$$
$$14 + 30 \to$$

$$(\lambda y.\, 10\cdot y)\,((\lambda x.\, x+x)(3)) \to (\lambda y.\, 10y)(3+3) \to (\lambda y.\, 10y)\, 6$$
$$\downarrow$$
$$10((\lambda x.\, x+x)(3))$$
$$\downarrow$$
$$10(3+3)$$
$$\downarrow$$
$$10\cdot 6$$
$$\downarrow$$
$$60$$

$\&$ $\vee$ $\to$ $\leftrightarrow$ $\neg$

Se $P=0$,

$$\underbrace{\underbrace{\neg \underbrace{\neg \underbrace{P}_{0}}_{1}}_{0} \to \underbrace{P}_{0}}_{1}$$

Se $P=1$,

$$\underbrace{\underbrace{\neg \underbrace{\neg \underbrace{P}_{1}}_{0}}_{1} \to \underbrace{P}_{1}}_{1}$$

Se $P=0$,

$\neg\neg P \to P$

0 1 0 0

1

Se $P=1$,

$\neg\neg P \to P$

1 0 1 1

1

$\neg\neg P \to P$

É UMA <u>TAUTOLOGIA</u>

P	Q	$P \vee Q \to P \& Q$
0	0	0 0 0 $\boxed{1}$ 0 0 0
0	1	0 1 1 $\boxed{0}$ 0 0 1
1	0	1 1 0 $\boxed{0}$ 1 0 0
1	1	1 1 1 $\boxed{1}$ 1 1 1

$\forall a,b \in \mathbb{R}.(a+b)(a-b)=a^2-b^2$

$P \vee Q \to P \& Q$

<u>NÃO</u> É UMA TAUTOLOGIA.

$$H = \begin{array}{ccc} & \cdot & \\ \cdot & & \cdot \\ \cdot & & \cdot \end{array} = \left\{\begin{array}{ccc} & (1,2), & \\ (0,1), & & (2,1) \\ (0,0), & & (2,0) \end{array}\right\}$$

$$r_H = \begin{array}{ccc} & 1 & \\ 2 & & 3 \\ 4 & & 5 \end{array} \qquad r_H((0,1)) = 2 \qquad r_H^{-1}(2) = (0,1)$$

$$\left\{\begin{array}{cccccc} 32 & & & & & \\ & 22 & & & & \\ & 22 & & 12 & & \\ 21 & & 11 & & 02 & \\ & 10 & & 01 & & \\ & & 00 & & & \end{array}\right\} = H'$$

$$\begin{array}{cc} & 0 \\ 1 & 0 \\ 1 & 1 \end{array} \multimap 21 \in H'$$

$$\| \quad P : \{1,2,3,4,5\} \multimap \{0,1\}$$

$$P = \begin{array}{ccc} & P_1 & \\ P_2 & & P_3 \\ P_4 & & P_5 \end{array} \qquad P = \left\{\begin{array}{cc} & (1,0), \\ (2,1), & (3,0), \\ (4,1), & (5,1) \end{array}\right\} \qquad P' = \left\{\begin{array}{cc} & ((1,2),0), \\ ((2,1),1), & ((3,0),0), \\ ((4,1),1), & ((5,1),1) \end{array}\right\}$$

$$P : \{1,2,3,4,5\} \multimap \{0,1\}$$

$$P' : H \multimap \{0,1\}$$

$$(\ell, r) \;\longmapsto\; \begin{array}{ccc} & \ell \geq 3 & \\ \ell \geq 2 & & r \geq 2 \\ \ell \geq 1 & & r \geq 1 \end{array}$$

$$(\ell, r) \;\xmapsto{f}\; \left\{\begin{array}{cc} & (1, \ell \geq 3), \\ (2, \ell \geq 2), & (3, r \geq 2), \\ (4, \ell \geq 1), & (5, r \geq 1) \end{array}\right\}$$

$$f : \mathbb{N}^2 \multimap (\{1,2,3,4,5\} \multimap \{0,1\})$$

$$(\ell, r) \mapsto \left\{\begin{array}{cc} & (1, \ell \geq 3), \\ (2, \ell \geq 2), & (3, r \geq 2), \\ (4, \ell \geq 1), & (5, r \geq 1) \end{array}\right\}$$

$$g : (\{1,2,3,4,5\} \multimap \{0,1\}) \multimap \mathbb{N}^2$$

$$P \mapsto (P_4 + P_2 + P_1, P_5 + P_3)$$

$$\left\{\begin{array}{cc} & (1, P_1), \\ (2, P_2), & (3, P_3), \\ (4, P_4), & (5, P_5) \end{array}\right\} \mapsto (P_4 + P_2 + P_1, P_5 + P_3)$$

$$\begin{array}{ccc} & P_1 & \\ P_2 & & P_3 \\ P_4 & & P_5 \end{array} \mapsto (P_4 + P_2 + P_1, P_5 + P_3)$$

$$f((1,2)) = \left\{\begin{array}{cc} & (1,0), \\ (2,0), & (3,1), \\ (4,1), & (5,1) \end{array}\right\} = \begin{array}{cc} & 0 \\ 0 & 1 \\ 1 & 1 \end{array}$$

$$\begin{array}{c} \mathbb{N}^2 \\ f \big\downarrow \big\uparrow g \\ \{1,2,3,4,5\} \multimap \{0,1\} \end{array} \qquad \begin{array}{c} (\ell, r) \\ f \big\downarrow \\ \left(\begin{array}{ccc} & \ell \geq 3 & \\ \ell \geq 2 & & r \geq 2 \\ \ell \geq 1 & & r \geq 1 \end{array}\right) \end{array} \qquad \begin{array}{c} (P_4 + P_2 + P_1, P_5 + P_3) \\ \big\uparrow g \\ \left(\begin{array}{ccc} & P_1 & \\ P_2 & & P_3 \\ P_4 & & P_5 \end{array}\right) \end{array}$$

$$f(2,2) = \left\{\begin{array}{cc} & (1,0), \\ (2,1), & (3,1), \\ (4,1), & (5,1) \end{array}\right\} = \begin{array}{cc} & 0 \\ 1 & 1 \\ 1 & 1 \end{array}$$

$$g\left(\begin{array}{cc} & 0 \\ 0 & 1 \\ 1 & 1 \end{array}\right) = (1,2) \qquad f\left(g\left(\begin{array}{cc} & 0 \\ 0 & 1 \\ 1 & 1 \end{array}\right)\right) = f((1,2)) = \begin{array}{cc} & 0 \\ 0 & 1 \\ 1 & 1 \end{array}$$

$$g\left(\begin{array}{cc} & 1 \\ 0 & 1 \\ 1 & 1 \end{array}\right) = (2,2) \qquad f\left(g\left(\begin{array}{cc} & 1 \\ 0 & 1 \\ 1 & 1 \end{array}\right)\right) = f((2,2)) = \begin{array}{cc} & 0 \\ 1 & 1 \\ 1 & 1 \end{array}$$

$\exists a : \{2,3,4\}.\ a^2 > 10$

$D = \{10, 20\}$

$g = \left\{ \begin{matrix} (1,10), \\ (2,20) \end{matrix} \right\}$

$h = \lambda p : A \times B.\ (g(\pi p), f(\pi' p))$

$$\cfrac{\cfrac{\cfrac{p}{\pi p}\,\pi \qquad \cancel{g}}{g(\pi p)}\,\mathrm{app} \qquad \cfrac{\cfrac{p}{\pi' p}\,\pi' \qquad f}{f(\pi' p)}\,\mathrm{app}}{\cfrac{(g(\pi p), f(\pi' p))}{\lambda p : A \times B.\ (g(\pi p), f(\pi' p))}\,\lambda}\,\mathrm{pair}$$

$$\cfrac{\cfrac{\cfrac{p : A \times B}{\pi p : A} \qquad g : A \to D}{g(\pi p) : D} \qquad \cfrac{\cfrac{p : A \times B}{\pi' p : B} \qquad f : B \to C}{f(\pi' p) : C}}{\cfrac{(g(\pi p), f(\pi' p)) : D \times C}{(\lambda p : A \times B.\ (g(\pi p), f(\pi' p))) : A \times B \to D \times C}}$$

$$\cfrac{\cfrac{P \,\&\, Q}{P} \qquad \cfrac{\cfrac{P \,\&\, Q}{Q} \qquad Q \to R}{R}}{P \,\&\, R}$$

F e g SÃO INVERSAS UMA DA OUTRA EM "VALORES DE VERDADE ESTÁVEIS" E "ELEMENTOS DE H".

$$\begin{tikzcd} \mathbb{N}^2 \arrow[d, shift right, "f"'] \\ \{1,2,3,4,5\} \to \{0,1\} \arrow[u, shift right, "g"'] \end{tikzcd}$$

$$(l, r) \overset{f}{\longmapsto} \begin{pmatrix} & l \geq 3 & \\ l \geq 2 & & r \geq 2 \\ l \geq 1 & & r \geq 1 \end{pmatrix} \qquad (p_4 + p_2 + p_1,\ p_5 + p_3) \overset{g}{\longleftarrow\!\shortmid} \begin{pmatrix} & p_1 & \\ p_2 & & p_3 \\ p_4 & & p_5 \end{pmatrix}$$

$$\begin{tikzcd} & \{1,2\} \arrow[dl, "f"'] \arrow[d] \arrow[dr, "g"] & \\ \{3,4\} & \{3,4\} \times \{5,6\} \arrow[l, "\pi"'] \arrow[r, "\pi'"] & \{5,6\} \end{tikzcd}$$

$f : B \to C$

$f \in (B \to C)$