$\boxed{\text{LA 22/AGOSTO/2017}}$

a) $\underbrace{\underbrace{2\cdot\underbrace{(3+4)}_{7}}_{14}+\underbrace{5\cdot 6}_{30}}_{44}$

$$\begin{tikzcd}
2\cdot(3+4)+5\cdot 6 \arrow[r] \arrow[d] & 2\cdot(3+4)+30 \arrow[d] \\
2\cdot 7+5\cdot 6 \arrow[r] \arrow[d] & 2\cdot 7+30 \arrow[d] \\
14+5\cdot 6 \arrow[r] & 14+30 \arrow[d] \\
 & 44
\end{tikzcd}$$

b) $\underbrace{\underbrace{2+3}_{5}+4}_{9}$

$$\begin{tikzcd}
2+3+4 \arrow[r] \arrow[d] & 2+7 \arrow[d] \\
5+4 \arrow[r] & 9
\end{tikzcd}$$

$2+3+4=2+7$
$=9$

a) $\underbrace{\underbrace{2\cdot\underbrace{(3+4)}_{7}}_{14}+\underbrace{5\cdot 6}_{30}}_{44}$

b) $\underbrace{\underbrace{2+3}_{5}+4}_{9}$

$$\begin{tikzcd}
2+3+4 \arrow[r] \arrow[d] & 2+7 \arrow[d] \\
5+4 \arrow[r] & 9
\end{tikzcd}$$

$2+3+4=2+7=9$

c) $2+3+4+5$

$$\begin{tikzcd}
2\cdot(3+4)+5\cdot 6 \arrow[r] \arrow[d] & 2\cdot(3+4)+30 \arrow[d] \\
2(7)+5\cdot 6 \arrow[r] \arrow[d] & 2\cdot(7)+30 \arrow[d] \\
14+5\cdot 6 \arrow[r] & 14+30 \arrow[d] \\
 & 44
\end{tikzcd}$$

$\underbrace{\underbrace{2\cdot\underbrace{(3+4)}_{7}}_{14}+\underbrace{5\cdot 6}_{30}}_{44}$

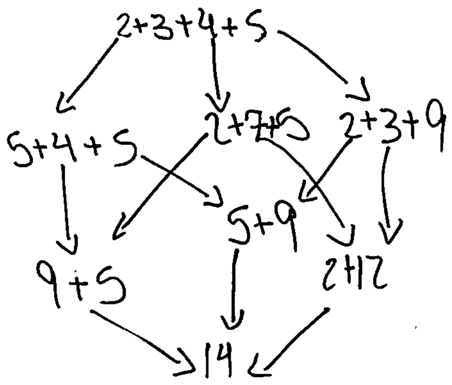
$$\begin{tikzcd}
2\cdot(3+4)+5\cdot 6 \arrow[r] \arrow[d] & 2\cdot(3+4)+30 \arrow[d] \\
2(7)+5\cdot 6 \arrow[r] \arrow[d] & 2(7)+30 \arrow[d] \\
14+5\cdot 6 \arrow[r] & 14+30 \arrow[d] \\
 & 44
\end{tikzcd}$$

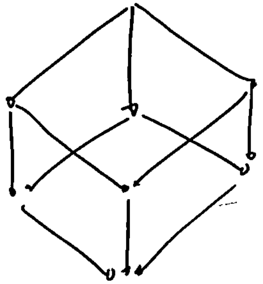
$\underbrace{\underbrace{2+3}_{5}+4}_{9}$

$$\begin{tikzcd}
2+3+4 \arrow[r] \arrow[d] & 2+7 \arrow[d] \\
5+4 \arrow[r] & 9
\end{tikzcd}$$

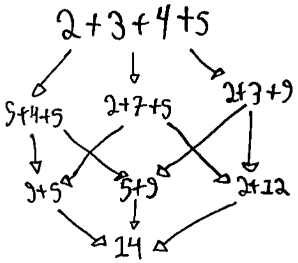
LA 22/AGOSTO/2017

c) $\underbrace{\underbrace{2+3}_{5}+\underbrace{4+5}_{9}}_{14}$

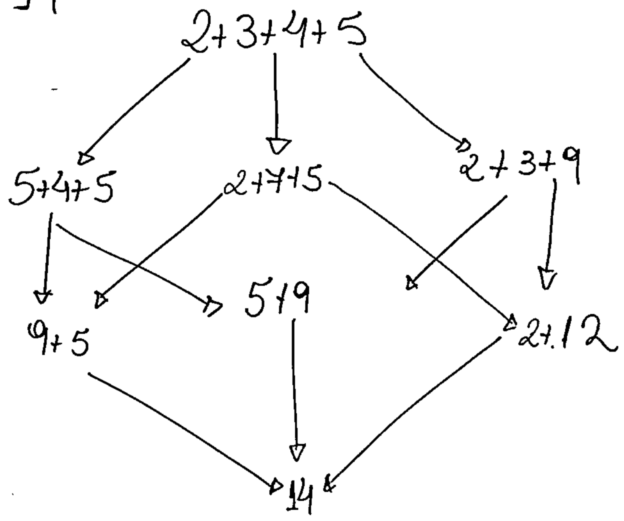




c) $\underbrace{\underbrace{2+3}_{5}+\underbrace{4+5}_{9}}_{14}$



$\underbrace{\underbrace{2+\underbrace{3+4}_{7}}_{9}+5}_{14}$



LA 22/AGOSTO/2017

O QUE ACONTECERIA SE A GENTE PERMITISSE ALGO COMO

$$\sum_{i \in A} f(i)\ ?$$

POR EXEMPLO,

$$\sum_{i \in \{2,3,5\}} i = ?$$

$$\underbrace{\underbrace{2+3}_{5}+5}_{10}$$

$$\sum_{i \in \{2,3,3,5\}} i = ?$$

$$\underbrace{\underbrace{2+3}_{5}+\underbrace{3+5}_{8}}_{13}$$

$$\{2,3,5\} = \{2,3,3,5\}$$

mas

$$\sum_{i \in \{2,3,5\}} i \neq \sum_{i \in \{2,3,3,5\}} i\ !!$$

$$\sum_{i=2}^{5} i \longrightarrow \sum_{i \in \{2,3,4,5\}} i \longrightarrow 2+3+4+5$$

Exercícios:

a) $(\lambda a.\,10a)(2+3)$

b) $(\lambda a.\,10a)((\lambda b.\,b+4)(3))$

c) $((\lambda a.\,(\lambda b.\,10a+b))(3))(4)$

b)

$(\lambda a.\,10a)((\lambda b.\,b+4)(3))$

$\downarrow$

$(\lambda a.\,10a)((b+4)[b:=3])$

$\downarrow$

$(\lambda a.\,10a)\ (3+4)$

Seja $h = (\lambda a.\,10a)$

$$\begin{array}{ccc} h(3+4) & \longrightarrow & h(7) \\ \downarrow & & \downarrow \\ (\lambda a.\,10a)(3+4) & \longrightarrow & (\lambda a.\,10a)(7) \\ \downarrow & & \downarrow \\ (10a)[a:=3+4] & \longrightarrow & (10a)[a:=7] \\ \downarrow & & \downarrow \\ 10(3+4) & \longrightarrow & 10.7 \\ & & \downarrow \\ & & 70 \end{array}$$

a) $(\lambda a.\,10a)\ (2+3)$

$$\begin{array}{ccc} h(2+3) & \longrightarrow & h(5) \\ \downarrow & & \downarrow \\ (\lambda a.\,10a)(2+3) & \longrightarrow & (\lambda a.\,10)(5) \\ \downarrow & & \downarrow \\ (10a)[a:=2+3] & \longrightarrow & (10a)[a:=5] \\ \downarrow & & \downarrow \\ (10.(2+3)) & \longrightarrow & 10.5 \\ & & \downarrow \\ & & 50 \end{array}$$

b)

$$\underbrace{\underbrace{\underbrace{(\lambda a.\,10a)(\underbrace{\underbrace{\underbrace{(\lambda b.\,b+4)(3)}_{(b+4)[b:=3]}}_{3+4}}_{7})}_{(10a)[a:=7]}}_{10.7}}_{70}$$

$$\underbrace{\underbrace{(\lambda a.\,10a)(\underbrace{\underbrace{(\lambda b.\,b+4)(3)}_{3+4}}_{7})}_{10.7}}_{70}$$

LA 22/AGOSTO/2017

O QUE ACONTECERIA SE A GENTE PERMITISSE ALGO COMO

$$\sum_{i \in A} f(i) ?$$

POR EXEMPLO,

$$\sum_{i \in \{2,3,5\}} i = ?$$

$$\underbrace{\underbrace{2+3}_{5}+5}_{10}$$

$$\sum_{i \in \{2,3,3,5\}} i = ?$$

$$\underbrace{\underbrace{2+3}_{5}+\underbrace{3+5}_{8}}_{13}$$

$$\{2,3,5\} = \{2,3,3,5\}$$

MAS

$$\sum_{i \in \{2,3,5\}} i \neq \sum_{i \in \{2,3,3,5\}} i \quad !!$$

$$\sum_{i=2}^{5} i \longrightarrow \sum_{i \in \{2,3,4,5\}} i \longrightarrow 2+3+4+5$$

EXERCÍCIOS:

a) $(\lambda a.\, 10a)(2+3)$

b) $(\lambda a.\, 10a)((\lambda b.\, b+4)(3))$

c) $((\lambda a.\,(\lambda b.\, 10a+b))(3))(4)$

d) $((\lambda f.\,(\lambda a.\, f(f(a)))(\lambda x.\, 10x))(7)$

c) $$\underbrace{((\lambda a.(\lambda b.10a+b))(3))}(4)$$

$$\underbrace{(\lambda b.\,10.3+b)(4)}$$

$$\underbrace{10.3+4}$$

$$\underbrace{30+4}$$

$$34$$

d) $$(\underbrace{(\lambda f.(\lambda a. f(f(a)))(\lambda x.10x)})(7)$$

$$\underbrace{((\lambda f.(\lambda a.\ f\ (\ f\ (a))))(\lambda x.10x))}(7)$$

$$\underbrace{((\lambda a.\ f\ (f\ (a)))\ (\lambda x.10x)[f := \lambda x.10x]}$$

$$\underbrace{((\lambda a.(\lambda x.10x)\ ((\lambda x.10x)\ (a))[a := 7]}$$

$$\underbrace{((\lambda x.10x)((\lambda x.10x).(7))}$$

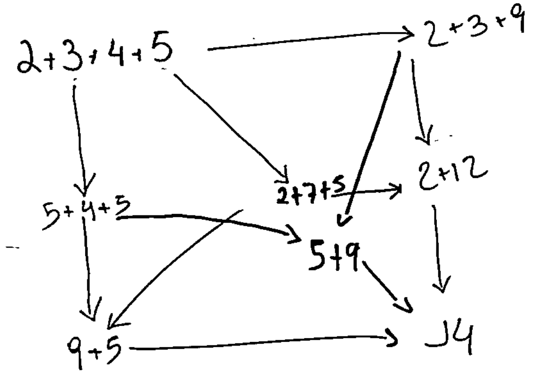
$$\underbrace{((\lambda x.10x)(10.7))}$$

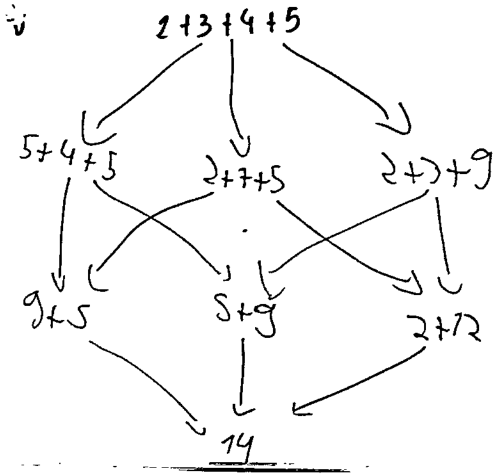
$$\underbrace{((\lambda x.10x)\ (70))}$$

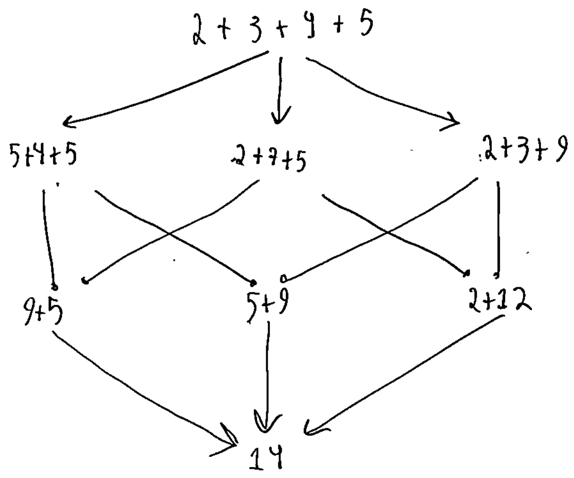
$$\underbrace{(10.70)}$$

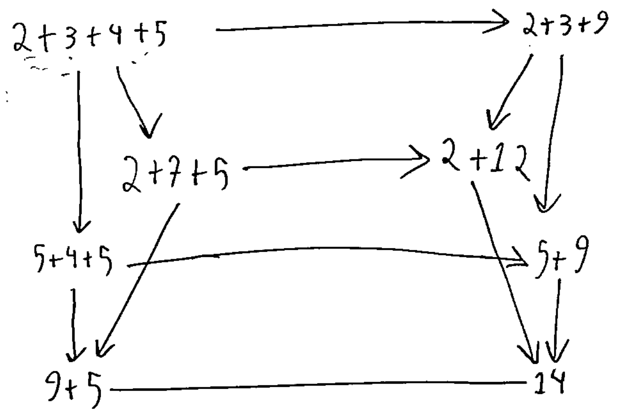
$$700$$

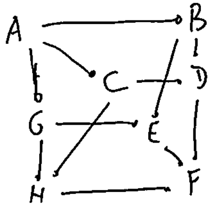
LA 29/AGO/2017











$\boxed{\text{LA 29/AGO/2017}}$

PRINCÍPIO:

TODAS AS SEQUÊNCIAS DE REDUÇÃO "CONVERGEM".

$$(2+3)\cdot(4+5) \to \dots \to 45$$

ESSE PRINCÍPIO VAI VALER SEMPRE.

$$\sum_{i=2}^{5} i^3 \to 2^3+3^3+4^3+5^3 \to \dots$$

$$\sum_{i \text{ in } (2,3,4,5)} i^3$$

← IDEIA NOVA

$$\sum_{i\in\{2,3,4,5\}} i^3$$

$$\sum_{i\in\{2,3,3,4,5\}} i^3$$

$$2^3+3^3+3^3+4^3+5^3 \to \dots$$

$$(a+b)(a-b) \to a^2-b^2$$

EXERCÍCIOS:

a) $(\lambda a.\,10a)(2+3)$

b) $\underbrace{(\lambda a.\,10a)}_{\alpha}\underbrace{(\underbrace{(\lambda b.\,b+4)}_{\beta}(3))}_{\gamma}$

c) $\underbrace{(\underbrace{(\lambda a.\,(\underbrace{\lambda b.\,10a+b}_{\beta}))}_{\alpha}(3))(4)}_{\gamma}$

d) $\underbrace{(\underbrace{(\lambda f.\,(\underbrace{\lambda a.\,f(f(a))}_{\alpha}))}_{\beta}\underbrace{(\lambda x.\,10x)}_{\gamma})(7)}_{\delta}$

C) $((\lambda a.\,(\lambda b.\,10a+b))(3))(4)$

$$\underbrace{(\underbrace{(\lambda a.\,(\underbrace{\lambda b.\,10a+b}_{\beta}))}_{\alpha}(3))(4)}_{\gamma}$$

$$\gamma(4) \to (\alpha(3))(4)$$

$$\begin{aligned}
\gamma(4) &= (\alpha(3))(4) \\
&= ((\lambda a.\,\beta)(3))(4) \\
&= (\beta[a:=3])(4) \\
&= ((\lambda b.\,10a+b)[a:=3])(4) \\
&= ((\lambda b.\,10\cdot 3+b))(4) \\
&= (\lambda b.\,30+b)(4) \\
&\overset{ok}{=} (30+b)[b:=4] \\
&\overset{ok}{=} (30+4) \\
&\overset{ok}{=} (34)
\end{aligned}$$

LA 5/set/2017

$A = \{1,2\}$

$B = \{3,4\}$

$C = \{30,40\}$

$D = \{10,20\}$

$f = \{(3,30),(4,40)\}$

$g = \{(1,10),(2,20)\}$

a) $A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

b) $A \times D = \{(1,10),(1,20),(2,10),(2,20)\}$

$A \to D = \left\{ \begin{Bmatrix}(1,10)\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,10)\\(2,20)\end{Bmatrix}, \begin{Bmatrix}(1,20)\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,20)\\(2,20)\end{Bmatrix} \right\}$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(\pi p, f(\pi' p))}\text{pair} \qquad \dfrac{\dfrac{(1,3)}{1}\pi \qquad \dfrac{\dfrac{(1,3)}{3}\pi' \quad f}{30}\text{app}}{(1,30)}\text{pair}$$

$$\dfrac{\dfrac{\dfrac{p}{\pi p}\pi \quad g}{g(\pi p)} \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}}{(g(\pi p), f(\pi' p))}\text{pair}$$

c) $p = (1,3)$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(\pi p, f(\pi' p))}\text{pair}$$

$(1,30)$

$p = (1,4)$

$$\dfrac{\dfrac{1,4}{1}\pi \qquad \dfrac{\dfrac{1,4}{4}\pi' \quad f}{40}\text{app}}{(1,40)}\text{pair}$$

$p = (2,3)$

$$\dfrac{\dfrac{(2,3)}{2}\pi \qquad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\text{app}}{(2,30)}\text{pair}$$

$p = (2,4)$

$$\dfrac{\dfrac{(2,4)}{2}\pi \qquad \dfrac{\dfrac{(2,4)}{4}\pi' \quad f}{40}\text{app}}{(2,40)}\text{pair}$$

d) $(\lambda p : A\times B.(\pi p, f(\pi' p))) = \{((1,3),(1,30)), ((1,4),(1,40)), ((2,3),(2,30)), ((2,4),(2,40))\}$ (IN, OUT)

$\lambda p : A \times B$

e) Yes!

$A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

$A \to D = \left\{ \begin{Bmatrix}(1,10),\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,10),\\(2,20)\end{Bmatrix}, \begin{Bmatrix}(1,20),\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,20),\\(2,20)\end{Bmatrix} \right\}$

$(\pi p, f(\pi' p)) =$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(\pi p, f(\pi' p))}\text{pair}$$

$$\dfrac{\dfrac{(1,3)}{1}\pi \qquad \dfrac{\dfrac{(1,3)}{3}\pi' \quad \{(3,30),(4,40)\}}{30}\text{app}}{(1,30)}\text{pair} \qquad \dfrac{\dfrac{(1,4)}{1}\pi \qquad \dfrac{\dfrac{(1,4)}{4}\pi' \quad f}{40}\text{app}}{(1,40)}\text{pair}$$

$$\dfrac{\dfrac{(2,3)}{2}\pi \qquad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\text{app}}{(2,30)}\text{pair} \qquad \dfrac{\dfrac{(2,4)}{2}\pi \qquad \dfrac{\dfrac{(2,4)}{4}\pi' \quad f}{40}\text{app}}{(2,40)}\text{pair}$$

LA 5/set/2017

$A = \{1,2\}$

$B = \{3,4\}$

$C = \{30,40\}$

$D = \{10,20\}$

$f = \{(3,30),(4,40)\}$

$g = \{(1,10),(2,20)\}$

$D \times C = \{(10,30),(10,40),(20,30),(20,40)\}$

a) $A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

b) $A \times D = \{(1,10),(1,20),(2,10),(2,20)\}$

$A \to D = \{\{(1,10),(2,10)\},\{(1,10),(2,20)\},\{(1,20),(2,10)\},\{(1,20),(2,20)\}\}$

$A \times D \to D \times C = \{$

$$\dfrac{\dfrac{p}{\pi p}\pi \quad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(\pi p, f(\pi' p))}\text{pair}
\qquad
\dfrac{\dfrac{(1,3)}{1}\pi \quad \dfrac{\dfrac{(1,3)}{3}\pi' \quad f}{30}\text{app}}{(1,30)}\text{pair}$$

$$\dfrac{\dfrac{\dfrac{p}{\pi p}\pi \quad g}{g(\pi p)}\text{app} \quad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(g(\pi p), f(\pi' p))}\text{pair}$$

$(20,30)$

1,4

10,

F

$$\dfrac{\dfrac{\dfrac{p}{\pi p}\pi \quad g}{g(\pi p)}\text{app} \quad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(g(\pi p), f(\pi' p))}\text{pair}$$

$(10,30)$

$p = (2,3)$

$$\dfrac{\dfrac{\dfrac{(2,3)}{2}\pi \quad g}{20}\text{app} \quad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\text{app}}{(20,30)}\text{pair}$$

$p = (2,3)$

$$\dfrac{\dfrac{(2,3)}{2}\pi \quad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\text{app}}{(2,30)}\text{pair}$$

$p = (2,4)$

$$\dfrac{\dfrac{(2,4)}{2}\pi \quad \dfrac{\dfrac{(2,4)}{4}\pi' \quad f}{40}\text{app}}{(2,40)}\text{pair}$$

H) $(\lambda p : A \times B . (g(\pi p), f(\pi' p)))$

$\{((1,3),(10,30)), ((1,4),(10,40)), ((2,3),(20,30)), ((2,4),(20,40))\}$

False

D) $(\lambda p : A \times B . (\pi p, f(\pi' p))) = \{$ in out $((1,3),(1,30)), ((1,4),(1,40)), ((2,3),(2,30)), ((2,4),(2,40))\}$

C) Yes!

$\lambda p : A \times B$

$A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

$A \to D = \{\{(1,10),(2,10)\},\{(1,10),(2,20)\},\{(1,20),(2,10)\},\{(1,20),(2,20)\}\}$

$(\pi p, f(\pi' p)) =$

$\lambda p : A \times B . (\pi p, f(\pi' p)) =$

$\{((1,3),(1,30)), ((1,4),(1,40)), ((2,3),(2,30)), ((2,4),(2,40))\}$

$$\dfrac{\dfrac{p}{\pi p}\pi \quad \dfrac{\dfrac{p}{\pi'}\pi' \quad f}{f(\pi' p)}\text{app}}{(\pi p, f(\pi' p))}\text{pair}$$

LA 5/SET/2017

$f : A \to B$
$g : B\times C \to D$
$h : B \to (C \to D)$
$k : D \to E$

(a) $$\underbrace{\lambda \underbrace{p}_{:A\times C} : A\times C . \underbrace{(\underbrace{f(\underbrace{\pi p}_{:A})}_{:B}, \underbrace{\pi' p}_{:C})}_{:B\times C}}_{A\times C \to B\times C}$$

(b) $$\underbrace{\lambda \underbrace{q}_{:B\times C} : B\times C . \underbrace{(\underbrace{h(\underbrace{\pi q}_{:B})}_{:C\to D})(\underbrace{\pi' q}_{:C})}_{:D}}_{:B\times C \to D}$$

(c) $$\underbrace{\lambda \underbrace{b}_{:B} : B . \underbrace{\lambda \underbrace{c}_{:C} : C . \underbrace{g\underbrace{(\underbrace{b}_{:B}, \underbrace{c}_{:C})}_{:B\times C}}_{:D}}_{:C\to D}}_{:B \to (C\to D)}$$

(d) $$\underbrace{\lambda \underbrace{\varphi}_{:C\to D} : C\to D . \lambda \underbrace{c}_{:C} : C . \underbrace{\kappa(\underbrace{\underbrace{\varphi}_{:C\to D} \underbrace{c}_{:C}}_{:D})}_{:E}}_{:C\times E}$$
$$:(C\to D)\to C\times E$$

$\varphi \; \infty$

$2 \in \{2,3,4\}$
$2 \in \{1,2,3\}$

$$\frac{p : A\times C}{\pi p : A}$$

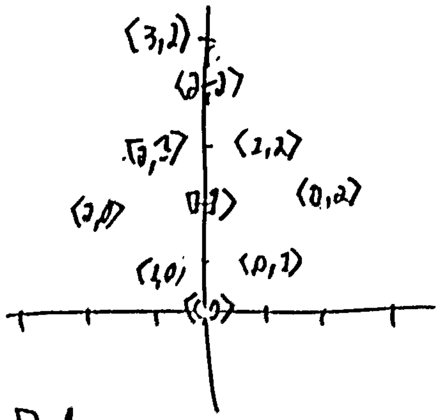
a) $$(\times C)f := \underbrace{\lambda \underbrace{p}_{A\times C} : A\times C . \underbrace{(\underbrace{\underbrace{f}_{A\to B}(\underbrace{\pi \underbrace{p}_{:A\times C}}_{:A})}_{:B}, \underbrace{\pi' \underbrace{p}_{:A\times C}}_{:C})}_{B\times C}}_{:A\times C \to B\times C}$$

b) $$h^{\flat} := \underbrace{\lambda q : B\times C . \underbrace{(\underbrace{\underbrace{h}_{:B\to(C\to D)}(\underbrace{\pi \underbrace{q}_{:B\times C}}_{:B})}_{:C\to D})(\underbrace{\pi' \underbrace{q}_{:B\times C}}_{:C})}_{:D}}_{:B\times C \to D}$$

c) $$g^{\#} := \underbrace{\lambda \underbrace{b}_{:B} : B . \underbrace{\lambda \underbrace{c}_{:C} : C . \underbrace{\underbrace{g}_{:B\times C\to D}\underbrace{(\underbrace{b}_{:B}, \underbrace{c}_{:C})}_{:B\times C}}_{:D}}_{:C\to D}}_{:B \to (C\to D)}$$

LA 12/SET/2017

HOJE: VAMOS INTERROMPER A PROGRAMAÇÃO NORMAL (λ-CÁLCULO TIPADO) PRA COMEÇAR A VER A RELAÇÃO ENTRE λ-CÁLCULO E LÓGICA!



a) $\lambda \ell_n : B.\ell$

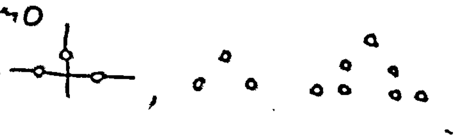
PLANAR HEYTING ALGEBRAS FOR CHILDREN

→ FIRST PAPER

→ SEÇÕES:

1. POSITIONAL NOTATIONS

ESCREVA COMO CONJUNTO:



2. ZDAGs

SEJA H =

REPRESENTE (H, BPM(H)) COMO GRAFO DIRECIONADO E COMO PAR DE CONJUNTOS.

3: LR-COORDINATES

PARA CADA UM DESTES PONTOS DÊ AS COORDENADAS (x,y) DELE E REPRESENTE-OS EM $\mathbb{Z}^2$.

⟨0,0⟩, ⟨1,0⟩, ⟨2,0⟩, ⟨0,1⟩, ⟨0,3⟩, ⟨1,3⟩, ⟨-1,0⟩, ⟨0,-1⟩, ⟨-1,-1⟩.

EXERCÍCIO (GRANDE):

SEJA $B = \begin{array}{ccccc} & 32 & & & \\ & & 22 & & \\ & 21 & & 12 & \\ 20 & & 11 & & 02 \\ & 10 & & 01 & \\ & & 00 & & \end{array}$

CALCULE, E REPRESENTE EM NOTAÇÃO POSICIONAL QUANDO POSSÍVEL:

a) $\lambda lr{:}B.\ l$
b) $\lambda lr{:}B.\ r$
c) $\lambda lr{:}B.\ (l \leq 1)$
d) $\lambda lr{:}B.\ (r \geq 1)$
e) $\lambda lr{:}B.\ lr \leq 11$
f) $\lambda lr{:}B.\ lr \& 12$
g) $\lambda lr{:}B.\ \mathrm{valid}(\langle l+1, r\rangle)$
h) $\lambda lr{:}B.\ lr$ leftof 11
i) $\lambda lr{:}B.\ lr$ leftof 12
j) $\lambda lr{:}B.\ lr$ above 11
k) $\lambda lr{:}B.\ \mathrm{ne}(lr)$
l) $\lambda lr{:}B.\ \mathrm{nw}(lr)$
m) $20 \multimap 11$
n) $02 \multimap 11$
o) $22 \multimap 11$
p) $00 \multimap 11$
q) $\lambda lr{:}B.\ \neg lr$
r) $\lambda lr{:}B.\ \neg\neg lr$
s) $\lambda lr{:}B.\ (lr = \neg\neg lr)$

ⓐ $\lambda lr : B.\ l$

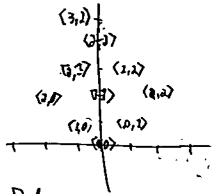
$$\begin{array}{ccccc} & 3 & & & \\ & & 2 & & \\ & 2 & & 1 & \\ 2 & & 1 & & 0 \\ & 1 & & 0 & \\ & & 0 & & \end{array}$$

lr	l	r	$(l \leq 1)$	$(r \geq 1)$	$(lr \leq 11)$
00	0	0	1	0	1
01	0	1	1	0	1
02	0	2	1	1	0
10	1	0	1	0	1
11	1	1	1	0	1
12	1	2	1	1	0
20	2	0	0	0	0
21	2	1	0	0	0
22	2	2	0	1	0
32	3	2	0	1	0

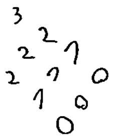
LA 12/SET/2017

lr	l	r	$l \le 1$	$r \ge 1$	$lr \le 11$
00	0	0	1	0	1
10	1	0	1	0	1
01	0	1	1	1	1
20	2	0	0	0	0
11	1	1	1	1	1
02	0	2	1	1	0
21	2	1	0	1	0
12	1	2	1	1	0
22	2	2	0	1	0
32	3	2	0	1	0

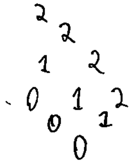
lr & 12	valid(⟨l+1, r⟩)	leftof 11
		10
00	1	10
10	1	11
01	1	20
20	0	20
11	1	11
12	1	11
12	0	21
12	1	11
	1	21
	0	31



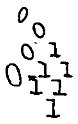
a) $\lambda lr:B.l$



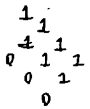
b)



c)

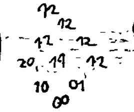


d)

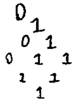


E)

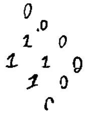
F)



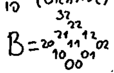
G)



h)



Exercício (grande):

Seja $B =$ 

Calcule, e represente em notação posicional quando possível:

a) $\lambda lr:B.\ l$
b) $\lambda lr:B.\ r$
c) $\lambda lr:B.\ (l \le 1)$
d) $\lambda lr:B.\ (r \ge 1)$
e) $\lambda lr:B.\ lr \le 11$
f) $\lambda lr:B.\ lr \& 12$
g) $\lambda lr:B.\ \mathrm{valid}(\langle l+1, r\rangle)$
h) $\lambda lr:B.\ lr$ leftof 11
i) $\lambda lr:B.\ lr$ leftof 12
j) $\lambda lr:B.\ lr$ above 11
k) $\lambda lr:B.\ \mathrm{ne}(lr)$
l) $\lambda lr:B.\ \mathrm{nw}(lr)$
m) $20 \to 11$
n) $02 \to 11$
o) $22 \to 11$
p) $00 \to 11$
q) $\lambda lr:B.\ \neg lr$
r) $\lambda lr:B.\ \neg\neg lr$
s) $\lambda lr:B.\ (lr = \neg\neg lr)$

00 · 12

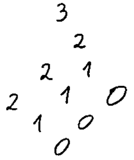
& g 00

&

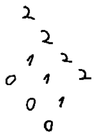
lr	l	r	lr leftof 11	valid(⟨l+1, r⟩)
00	0	0	0	1
01	0	1	0	1
02	0	2	0	1
10	1	0	1	1
11	1	1	1	1
12	1	2	0	1
20	2	0	1	0
21	2	1	1	0
22	2	2	0	1
32	3	2	0	0

$\Omega = B$

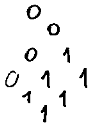
a) $\lambda lr:B.\ l$



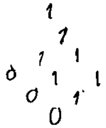
b) $\lambda lr:B.\ r$



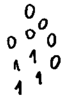
c) $\lambda lr:B.\ (l \le 1)$



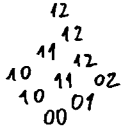
d) $\lambda lr:B\ (r \ge 1)$



e) $\lambda lr:B.\ lr \le 11$

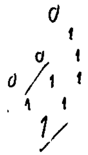


f) $\lambda lr:B.\ lr \,\&\, 12$

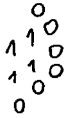


$l \ge l'$ & $r \le r'$

g) $\lambda lr:B.\ \mathrm{valid}(\langle l+1, r\rangle)$



h) $\lambda lr:B.\ lr$ leftof 11



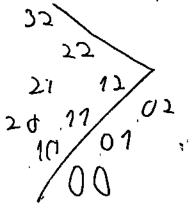
LA 26/set/2017

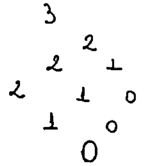
1) Positional notations
Escreva como conjunto:

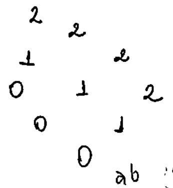
2) ZDAGs
Seja H = 
Represente (H, BPm(H)) como grafo direcionado e como par de conjuntos.

3) LR-coordinates
Para cada um destes pontos dê as coordenadas (x,y) dele e represente-o em $\mathbb{Z}^2$.
⟨0,0⟩, ⟨1,0⟩, ⟨2,0⟩, ⟨0,1⟩, ⟨0,3⟩, ⟨1,3⟩, ⟨-1,0⟩, ⟨0,-1⟩, ⟨-1,-1⟩.

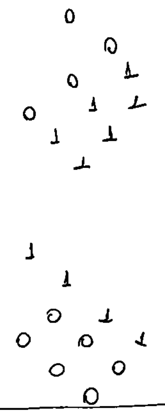
⟨1,3⟩ ⟨0,3⟩
⟨2,0⟩ ⟨1,0⟩ ⟨0,1⟩ ⟨0,0⟩ ⟨-1,0⟩ ⟨0,-1⟩ ⟨-1,-1⟩

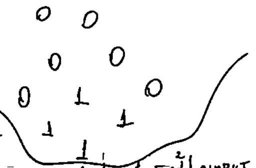
B = 

a) 

b) 

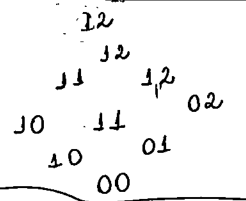
V
F

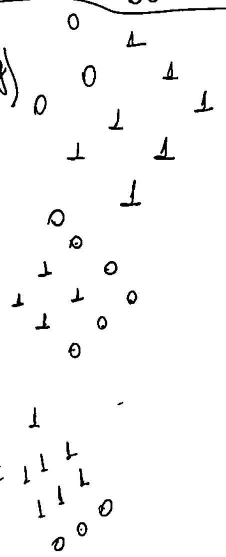
c) d) 

e) 

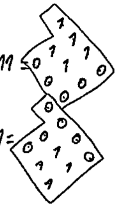
ab	a≥c	b≤d	OUTPUT
00	0	1	0
01	0	1	0
01	0	0	0
10	1	1	1
11	1	1	1
12	1	0	0
20	1	1	1
21	1	1	1
22	1	0	0
32	1	0	0

ab	a≥c	b≤d	OUTPUT
00	0	1	0
01	0	1	0
02	0	1	0
10	1	1	1
11	1	1	1
12	1	1	1
20	1	1	1
21	1	1	1
22	1	1	1
32	1	1	1

f) 

g) h) i) 

(a, b) leftof (c, d)
$a \geq c \wedge b \leq d$

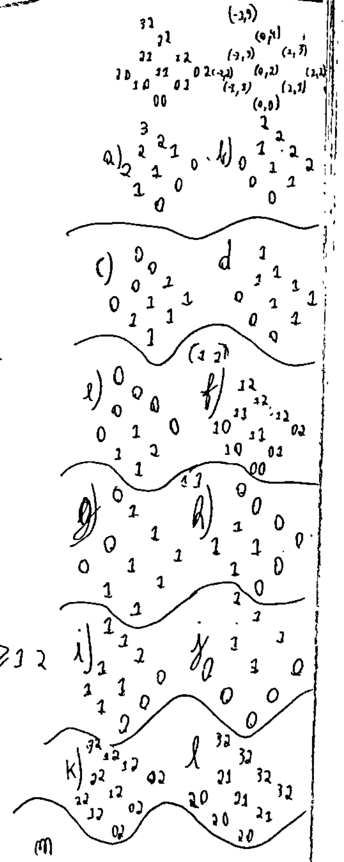
λ lr:B. lr above 11 = 

λ lr:B. lr below 11 =

(0, 0+1)

ne(00) = ne(01) = ne(02) 02
ne(01) = 02
ne(02) = 02
ne(10) = 12

≥ 12



LA 10/OUT/2017

ÁLGEBRAS DE HEYTING OBEDECEM ISSO AQUI:

$$\begin{tikzcd} P \arrow[d] \\ \top \end{tikzcd} \qquad \begin{tikzcd} & P \arrow[dl] \arrow[d] \arrow[dr] & \\ Q & Q\wedge R \arrow[l] \arrow[r] & R \end{tikzcd} \qquad \begin{tikzcd} P\wedge Q \arrow[r, leftarrow] \arrow[d] & P \arrow[d] \\ R \arrow[r, mapsto] \arrow[ur, phantom, "\leftrightarrow"] & Q\to R \end{tikzcd}$$

$$\begin{tikzcd} \bot \arrow[d] \\ P \end{tikzcd} \qquad \begin{tikzcd} P \arrow[r] \arrow[dr] & P\vee Q \arrow[d] & Q \arrow[l] \arrow[dl] \\ & R & \end{tikzcd}$$

$(P\le Q)\wedge(P\le R) \leftrightarrow (P\le(Q\wedge R))$ ①

$\lambda P.(P\le Q \wedge P\le R) = \lambda P.\ P\le Q\wedge R$ ②

$(P\wedge Q\le R) \leftrightarrow (P\le Q\to R)$ ③

$\lambda P.(P\wedge Q\le R) = \lambda P.(P\le(\underbrace{Q\to R}_{?}))$ ④

SEJA Ω =

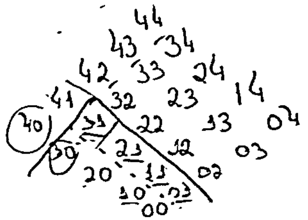
SE $Q=31$ E $R=12$,
ENCONTRE O ÚNICO VALOR PRO "?" QUE OBEDECE ISTO:

$\lambda P.(P\wedge Q\le R) = \lambda P.(P\le ?)$ $\quad ? = 14$

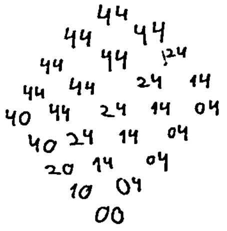
$\lambda P.((P\wedge \overset{31}{Q})\le \overset{12}{R})$

$\langle \min(a,3), \min(b,1)\rangle$

$\lambda P.(P\wedge Q)$



R	$\overset{31}{Q}\to R$
00	00
01	04
02	04
03	04
04	04
10	10
11	14
12	14
13	14
14	14
20	20
21	24
22	24
23	24
24	24
30	40
31	44
32	44
33	44
34	44
40	40
41	44
42	44
43	44
44	44



R	$\overset{31}{Q}\to R$
00	00
01	04
02	04
03	04
04	04
10	10
11	14
12	14
13	14
14	14
20	20
21	24
22	24
23	24
24	24
30	40
31	44
32	44
33	44
34	44
40	40
41	44
42	44
43	44
44	44

Q ↓R T → sup(Ω)
← me
→ mw
1 (c,d)

$Q=31 \quad R=12$

$(\lambda P.\ P\wedge Q) =$

$(\lambda P.\ P\wedge 31) =$

$= (\lambda P.(P\wedge \overset{Q}{31})\le \overset{R}{12})$

$= (\lambda P.(P\le ?))$ $\quad ?=14$

$Q\to R$

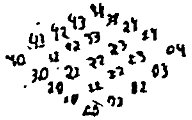
$22\to R = ?$

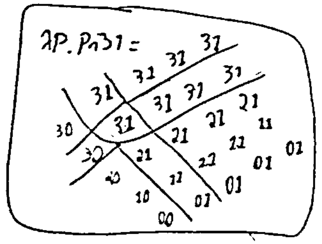
$Q\to R$

$22\to 12 = 14$

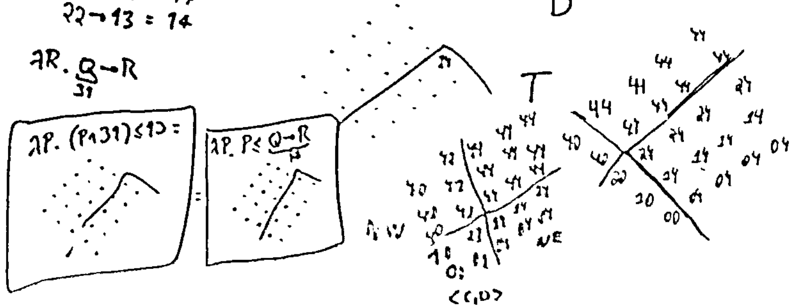
$22\to 13 = 14$

$\lambda R.\ \underset{31}{Q}\to R$





B



LA 10/out/2017

ÁLGEBRAS DE HEYTING
OBEDECEM ISSO AQUI:

$$
\begin{tikzcd}
P \arrow[d] \\
\top
\end{tikzcd}
\qquad
\begin{tikzcd}
 & P \arrow[dl] \arrow[d] \arrow[dr] & \\
Q & Q \land R \arrow[l] \arrow[r] & R
\end{tikzcd}
\qquad
\begin{tikzcd}
P \land Q \arrow[d] \arrow[r, phantom, "\leftrightarrow"] & P \arrow[d] \arrow[l, mapsto] \\
R \arrow[r, mapsto] & Q \to R
\end{tikzcd}
$$

$$
\begin{tikzcd}
\bot \arrow[d] \\
P
\end{tikzcd}
\qquad
\begin{tikzcd}
P \arrow[r] \arrow[dr] & P \lor Q \arrow[d] & Q \arrow[l] \arrow[dl] \\
 & R &
\end{tikzcd}
$$

$(P \leq Q) \land (P \leq R) \leftrightarrow (P \leq \underbrace{(Q \land R)}_{?})$ ①

$\lambda P.(P \leq Q \land P \leq R) = \lambda P.\ P \leq Q \land R$ ②

$(P \land Q \leq R) \leftrightarrow (P \leq Q \to R)$ ③

$\lambda P.(P \land Q \leq R) = \lambda P.(P \leq \underbrace{(Q \overset{B}{\to} R)}_{?})$ ④

Seja $\Omega =$ 

Se $Q = 31$ e $R = 12$,
encontre o único valor pro "?"
que obedece isto:

$\lambda P.(\underbrace{P \land Q}\leq R) = \lambda P.(P \leq ?)$

$(1\to)$ $P \leq Q \land P \leq R \to P \leq Q \land R$

$(1\leftarrow)$ $P \leq Q \land P \leq R \leftarrow P \leq Q \land R$

$(1\leftarrow_a)$ $P \leq Q \quad\quad \leftarrow P \leq Q \land R$

$(1\leftarrow_b)$ $\quad\quad P \leq R \leftarrow P \leq Q \land R$

$(3\to)$ $P \land Q \leq R \to P \leq Q \to R$

$(3\leftarrow)$ $P \land Q \leq R \leftarrow P \leq Q \to R$

$(1\to)$: $\dfrac{P \leq Q \quad P \leq R}{P \leq Q \land R}$

$(1\leftarrow_a)$: $\dfrac{P \leq Q \land R}{P \leq Q}$ $\quad$ $(1\leftarrow_b)$: $\dfrac{P \leq Q \land R}{P \leq R}$

R	$\overbrace{Q}^{31} \to R$
00	00
01	04
02	04
03	04
04	04
10	10
11	14
12	14
13	14
14	14
20	20
21	24
22	24
23	24
24	
30	40
31	44
32	44
33	44
34	44
40	40
41	44
42	44
43	44
44	

LA 17/out/2017

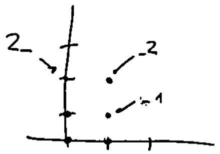
Hoje: topologias!

Estamos no Auditório...

Irene

$X\_ = (0\ X)$

$\_X = (1\ X)$



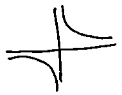
$(-\infty, 0]$ $[1, \infty)$

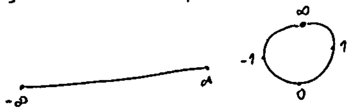
$(0,1)$

$(0, \infty)$

$\mathbb{R} = (-\infty, \infty)$

$\mathbb{R} \cup \{-\infty, +\infty\} = [-\infty, +\infty]$

$1/x =$ 



$(-\infty, 42]$ $(42, 99)$ $[99, +\infty)$

$(-\infty, \infty)$

$\emptyset$

$0\,{}_0\,0 \Rightarrow \emptyset$

$0\,{}_1\,0 \Rightarrow (0,1)$

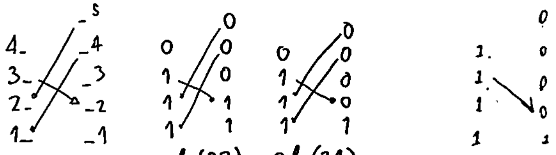
$0\,{}_1\,1 \Rightarrow (0,1) \cup [1,\infty) = (0,\infty)$

$1\,{}_1\,0 \Rightarrow (-\infty,0] \cup [0,1) = (-\infty,1)$

$= (-\infty, \infty)$

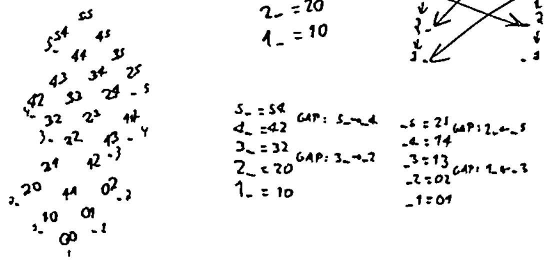
$1\,{}_1\,1 \Rightarrow$

$1\,{}_0\,0 = (-\infty, 0]$



pile(32) pile(31)

5_ = 54
4_ = 42
3_ = 32
2_ = 20
1_ = 10



5_ = 54 GAP: 5_→_4
4_ = 42
3_ = 32
2_ = 20 GAP: 3_→_2
1_ = 10

5 = 21 GAP: 2→_5
_4 = 14
_3 = 13
2 = 02 GAP: 1→_3
_1 = 01

LA 31/OUT/2017

$(2,4) \cap (3,5) = (3,4)$

$(2,4) \cap [3,5] = [3,4)$

$(2,4) \cup (3,5) = (2,5)$

$(2,4) \cup [3,5] = (2,5]$

$(\frac{1}{2},1) \cup (\frac{1}{3},1) \cup (\frac{1}{4},1) \cup \dots =$

$= (0,1)$

$[\frac{1}{2},1] \cup [\frac{1}{3},1] \cup [\frac{1}{4},1] \cup \dots = \quad (0,1]$

$(0,2) \cup (1,3) \cup (2,4) \cup (3,5) \cup \dots = ?$

$(0,+\infty)$

$(-1,\frac{1}{2}) \cap (-1,\frac{1}{3}) \cap (-1,\frac{1}{4}) \cap \dots = (-1,0]$

$(0,\frac{1}{2}) \cap (0,\frac{1}{3}) \cap (0,\frac{1}{4}) \cap \dots = \emptyset$

$\mathbb{R} \setminus (2,3) = (-\infty,2] \cup [3,+\infty)$

$\mathbb{R} \setminus [2,3] = (-\infty,2) \cup (3,+\infty)$

UM CONJUNTO $A \subset \mathbb{R}$
É FECHADO SE E SÓ SE
$\mathbb{R} \setminus A$ É ABERTO.

$\emptyset$ É ABERTO

$\{1,2,3,4\} \setminus \{3,4,5,6\} = \{1,2\}$

SEJA $V = \because$

ENTÃO $\mathcal{P}(V) = \left\{ \substack{00\\0}, \substack{00\\1}, \substack{01\\0}, \substack{01\\1}, \substack{10\\0}, \substack{10\\1}, \substack{11\\0}, \substack{11\\1} \right\}$

$\mathcal{O}(V) = \left\{ \substack{00\\0}, \substack{00\\1}, \substack{01\\1}, \substack{10\\1}, \substack{11\\1} \right\}$

$(V, \mathcal{P}(V))$,

$(V, \mathcal{O}(V))$ E

$\left(V, \left\{ \substack{00\\0}, \substack{11\\1} \right\}\right)$ SÃO ESPAÇOS TOPOLÓGICOS.

$P = \overset{\cdot}{0\,\cdot\,\cdot}$ $\quad Q = \overset{\cdot}{\cdot\,0\,\cdot}$

$Q \vee R = \overset{\cdot}{\cdot\,0\,0}$ $\quad R = \overset{\cdot}{\cdot\,\cdot\,0}$

$P \to (Q \vee R)$

$$\mathrm{int}\ \underbrace{\underbrace{\underbrace{P}_{\overset{0}{100}} \overset{m}{\to} \underbrace{(\underbrace{Q}_{\overset{0}{010}} \vee \underbrace{R}_{\overset{0}{001}})}_{\overset{0}{011}}}_{}}_{\overset{1}{0\,1\,1}}$$

$\overset{0}{0\,1\,1}$

$$\mathrm{int}\left(\underbrace{\underbrace{Q}_{\overset{\cdot}{\cdot\,0\,\cdot}} \overset{m}{\to} \underbrace{(\underbrace{R}_{} \vee \underbrace{P}_{})}_{\overset{\cdot}{\cdot\,\cdot\,0} \quad \overset{\cdot}{0\,\cdot\,\cdot}}}_{\overset{\cdot}{0\,\cdot\,0}}\right)$$

$\underbrace{\qquad}_{\overset{0}{0\,\cdot\,0}}$

$\overset{\cdot}{0\,\cdot\,0}$

$$\underbrace{\underbrace{\underbrace{S_P}_{\overset{\cdot}{\cdot\,0\,0}} \vee \underbrace{S_Q}_{\overset{\cdot}{0\,\cdot\,0}}}_{\overset{\cdot}{0\,0\,0}} \vee \underbrace{S_R}_{\overset{\cdot}{0\,0\,\cdot}}}_{\overset{\cdot}{0\,0\,0}}$$

$$\mathrm{int}\left(\underbrace{\underbrace{R}_{\overset{\cdot}{\cdot\,\cdot\,0}} \overset{m}{\to} \underbrace{(\underbrace{P}_{} \vee \underbrace{Q}_{})}_{\overset{\cdot}{0\,\cdot\,\cdot} \quad \overset{\cdot}{\cdot\,0\,\cdot}}}_{\overset{\cdot}{0\,0\,\cdot}}\right)$$

$\underbrace{\qquad}_{\overset{0}{0\,0\,\cdot}}$

$\overset{\cdot}{0\,0\,\cdot}$

LA 7/NOV/2017

PRÓXIMOS ASSUNTOS DO CURSO:
CATEGORIAS,
COMO "OMITIR VARIÁVEIS",
LINGUAGEM DE PROGRAMAÇÃO COM "λ".

$$\cfrac{\cfrac{\cfrac{p : A\times C}{\pi p : A}\pi \qquad f : A \to C}{f(\pi p) : B}\text{APP} \qquad \cfrac{p : A\times C}{\pi' p : C}\pi'}{\cfrac{(f(\pi p), \pi' p) : B\times C}{\lambda p : A\times C . (f(\pi p), \pi' p) : A\times C \to B\times C}\lambda}\text{PAIR}$$

p. 13:

$$\cfrac{f, p \vdash (\pi p, f(\pi' p))}{f \vdash \lambda p : A\times B . (\pi p, f(\pi' p))}\lambda$$

$$\cfrac{f, p \vdash (\pi p, f(\pi' p))}{p \vdash \lambda f : B\to C . (\pi p, f(\pi' p))}\lambda$$

$$\cfrac{f, p \vdash (\pi p, f(\pi' p))}{f, p \vdash \lambda d : D . (\pi p, f(\pi' p))}\lambda$$

$$\cfrac{\cfrac{\cfrac{q : B\times C}{\pi' q : C}\pi' \qquad \cfrac{\cfrac{q : B\times C}{\pi q : B}\pi \qquad h : B \to (C \to D)}{h(\pi q) : C \to D}\text{APP}}{h(\pi q)(\pi' q) : D}\text{APP}}{\lambda q : B\times C . h(\pi q)(\pi' q) : B\times C \to D}\lambda$$

$$\cfrac{\cfrac{\cfrac{Q\wedge R}{R} \qquad \cfrac{\cfrac{Q\wedge R}{Q} \qquad Q\to(R\to S)}{R\to S}}{S}}{Q\wedge R \to S}$$

(A, B, a, p), ONDE:
A É UM CONJUNTO,
B " " "
$a \in A$,
$p \in A\times B$.

3 - 4
-4

$\mathbb{N}$
A
LIST(A)
TREE(A)

$$\cfrac{p : P \qquad f : P \to Q}{fp : Q}$$

$$\cfrac{P\wedge Q \qquad Q\to R}{P\wedge R}$$

23 22 21 12 20 11 02 10 01 00

$P \to Q$

P

$A \to B$

$$\cfrac{P \quad G}{G \to R}$$

$$\cfrac{P \quad G}{P \& G}\&I \qquad \cfrac{P\& G}{P}\&E_1 \qquad \cfrac{P\& G}{G}\&E_2$$

$$\cfrac{\begin{matrix} P \quad [G]^1 \\ \vdots \\ R \end{matrix}}{G \to R}\to I$$