LA 22/AGOSTO/2017

a) $2.\underbrace{\underbrace{(3+4)}_{7}}_{14}+\underbrace{5.6}_{30}$ = 44

$$\begin{array}{ccc} 2.(3+4)+5.6 & \to & 2.(3+4)+30 \\ \downarrow & & \downarrow \\ 2.7+5.6 & \to & 2.7+30 \\ \downarrow & & \downarrow \\ 14+5.6 & \to & 14+30 \\ & & \downarrow \\ & & 44 \end{array}$$

b) $\underbrace{\underbrace{2+3}_{5}+4}_{9}$

$$\begin{array}{ccc} 2+3+4 & \to & 2+7 \\ \downarrow & & \downarrow \\ 5+4 & \to & 9 \end{array}$$

$2+3+4=2+7$
$=9$

a) $2.\underbrace{\underbrace{(3+4)}_{7}}_{14}+\underbrace{5.6}_{30}$ = 44

b) $\underbrace{\underbrace{2+3}_{5}+4}_{9}$

$$\begin{array}{ccc} 2+3+4 & \to & 2+7 \\ \downarrow & & \downarrow \\ 5+4 & \to & 9 \end{array}$$

$2+3+4=2+7=9$

c) $2+3+4+5$

$$\begin{array}{ccc} 2.(3+4)+5.6 & \to & 2.(3+4)+30 \\ \downarrow & & \downarrow \\ 2(7)+5.6 & \to & 2.(7)+30 \\ \downarrow & & \downarrow \\ 14+5.6 & \to & 14+30 \\ & & \downarrow \\ & & 44 \end{array}$$

$2\cdot\underbrace{\underbrace{(3+4)}_{7}}_{14}+\underbrace{5.6}_{30}$ = 44

$$\begin{array}{ccc} 2\cdot(3+4)+5.6 & \to & 2.(3+4)+30 \\ \downarrow & & \downarrow \\ 2(7)+5.6 & \to & 2(7)+30 \\ \downarrow & & \downarrow \\ 14+5.6 & \to & 14+30 \\ & & \downarrow \\ & & 44 \end{array}$$

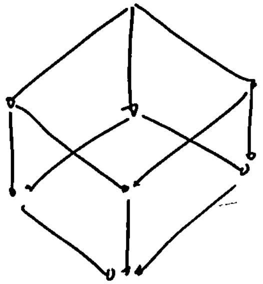
$\underbrace{\underbrace{2+3}_{5}+4}_{9}$

$$\begin{array}{ccc} 2+3+4 & \to & 2+7 \\ \downarrow & & \downarrow \\ 5+4 & \to & 9 \end{array}$$

LA 22/AGOSTO/2017

c) $\underbrace{\underbrace{2+3}_{5}+\underbrace{4+5}_{9}}_{14}$

$$\begin{tikzcd}
 & 2+3+4+5 \arrow[dl] \arrow[d] \arrow[dr] & \\
5+4+5 \arrow[d] \arrow[dr] & 2+7+5 \arrow[dl] \arrow[dr] & 2+3+9 \arrow[dl] \arrow[d] \\
9+5 \arrow[dr] & 5+9 \arrow[d] & 2+12 \arrow[dl] \\
 & 14 &
\end{tikzcd}$$



c) $\underbrace{\underbrace{2+3}_{5}+\underbrace{4+5}_{9}}_{14}$

$$\begin{tikzcd}
 & 2+3+4+5 \arrow[dl] \arrow[d] \arrow[dr] & \\
5+4+5 \arrow[d] \arrow[dr] & 2+7+5 \arrow[dl] \arrow[dr] & 2+3+9 \arrow[dl] \arrow[d] \\
9+5 \arrow[dr] & 5+9 \arrow[d] & 2+12 \arrow[dl] \\
 & 14 &
\end{tikzcd}$$

$\underbrace{\underbrace{2+\underbrace{3+4}_{7}}_{9}+5}_{14}$

$$\begin{tikzcd}
 & 2+3+4+5 \arrow[dl] \arrow[d] \arrow[dr] & \\
5+4+5 \arrow[d] \arrow[dr] & 2+7+5 \arrow[dl] \arrow[dr] & 2+3+9 \arrow[dl] \arrow[d] \\
9+5 \arrow[dr] & 5+9 \arrow[d] & 2+12 \arrow[dl] \\
 & 14 &
\end{tikzcd}$$

LA 22/AGOSTO/2017

O QUE ACONTECERIA SE A GENTE PERMITISSE ALGO COMO

$$\sum_{i\in A} f(i) \;?$$

POR EXEMPLO,

$$\sum_{i\in\{2,3,5\}} i = ?$$

$$\downarrow$$

$$\underbrace{\underbrace{2+3}_{5}+5}_{10}$$

$$\sum_{i\in\{2,3,3,5\}} i = ?$$

$$\downarrow$$

$$\underbrace{\underbrace{2+3}_{5}+\underbrace{3+5}_{8}}_{13}$$

$$\{2,3,5\} = \{2,3,3,5\}$$

mas

$$\sum_{i\in\{2,3,5\}} i \neq \sum_{i\in\{2,3,3,5\}} i \;!!$$

$$\sum_{i=2}^{5} i \longrightarrow \sum_{i\in\{2,3,4,5\}} i \longrightarrow 2+3+4+5$$

Exercícios:

a) $(\lambda a.\,10a)(2+3)$

b) $(\lambda a.\,10a)((\lambda b.\,b+4)(3))$

c) $((\lambda a.\,(\lambda b.\,10a+b))(3))(4)$

b)

$$(\lambda a.\,10a)((\lambda b.\,b+4)(3))$$
$$\downarrow$$
$$(\lambda a.\,10a)((b+4)[b:=3])$$
$$\downarrow$$
$$(\lambda a.\,10a)(3+4)$$

Seja $h = (\lambda a.\,10a)$

$$\begin{array}{ccc}
h(3+4) & \longrightarrow & h(7) \\
\downarrow & & \downarrow \\
(\lambda a.\,10a)(3+4) & \longrightarrow & (\lambda a.\,10a)(7) \\
\downarrow & & \downarrow \\
(10a)[a:=3+4] & \longrightarrow & (10a)[a:=7] \\
\downarrow & & \downarrow \\
10(3+4) & \longrightarrow & 10.7 \\
 & & \downarrow \\
 & & 70
\end{array}$$

a) $(\lambda a.\,10a)(2+3)$

$$\begin{array}{ccc}
h(2+3) & \longrightarrow & h(5) \\
\downarrow & & \downarrow \\
(\lambda a.\,10a)(2+3) & \longrightarrow & (\lambda a.\,10)(5) \\
\downarrow & & \downarrow \\
(10a)[a:=2+3] & \longrightarrow & (10a)[a:=5] \\
\downarrow & & \downarrow \\
(10.(2+3)) & \longrightarrow & 10.5 \\
 & & \downarrow \\
 & & 50
\end{array}$$

b)

$$\underbrace{\underbrace{\underbrace{(\lambda a.\,10a)\underbrace{\underbrace{\underbrace{((\lambda b.\,b+4)(3))}_{(b+4)[b:=3]}}_{3+4}}_{7}}_{(10a)[a:=7]}}_{10.7}}_{70}$$

$$\underbrace{\underbrace{(\lambda a.\,10a)\underbrace{\underbrace{((\lambda b.\,b+4)(3))}_{3+4}}_{7}}_{10.7}}_{70}$$

$\boxed{\text{LA 22/AGOSTO/2017}}$

O QUE ACONTECERIA SE A GENTE PERMITISSE ALGO COMO

$$\sum_{i \in A} f(i) \,?$$

POR EXEMPLO,

$$\sum_{i \in \{2,3,5\}} i = ? \qquad \sum_{i \in \{2,3,3,5\}} i = ?$$

$$\underbrace{\underbrace{2+3}_{5}+5}_{10} \qquad \underbrace{\underbrace{2+3}_{5}+\underbrace{3+5}_{8}}_{13}$$

$$\{2,3,5\} = \{2,3,3,5\}$$

MAS

$$\sum_{i \in \{2,3,5\}} i \neq \sum_{i \in \{2,3,3,5\}} i \;!!$$

$$\sum_{i=2}^{5} i \longrightarrow \sum_{i \in \{2,3,4,5\}} i \longrightarrow 2+3+4+5$$

EXERCÍCIOS:

a) $(\lambda a.\, 10a)(2+3)$

b) $(\lambda a.\, 10a)((\lambda b.\, b+4)(3))$

c) $((\lambda a.\, (\lambda b.\, 10a+b))(3))(4)$

d) $((\lambda f.\, (\lambda a.\, f(f(a)))(\lambda x.\, 10x))(7)$

c) $$\underbrace{\underbrace{\underbrace{\underbrace{(\underbrace{(\lambda a.(\lambda b.10a+b))(3)}(4)}_{(\lambda b.\,10.3+b)(4)}}_{10.3+4}}_{30+4}}_{34}$$

d) $$(\underbrace{(\lambda f.(\lambda a.f(f(a)))(\lambda x.10x)})(7)$$

$$\underbrace{((\lambda f.(\lambda a.\ f\ (\ f\ (a))))(\lambda x.10x))}(7)$$

$$\underbrace{((\lambda a.\ f\ (f\ (a)))\ (\lambda x.10x)[f := \lambda x.10x]}$$

$$\underbrace{((\lambda a.(\lambda x.10x)((\lambda x.10x)\ (a))[a := 7]}$$

$$\underbrace{((\lambda x.10x)((\lambda x.10x)(7))}$$

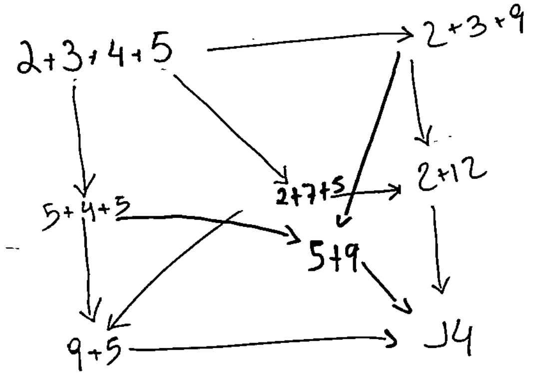
$$\underbrace{((\lambda x.10x)(10.7))}$$

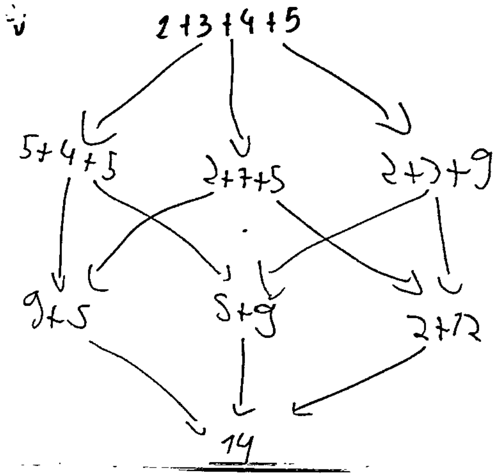
$$\underbrace{((\lambda x.10x)\ (70))}$$

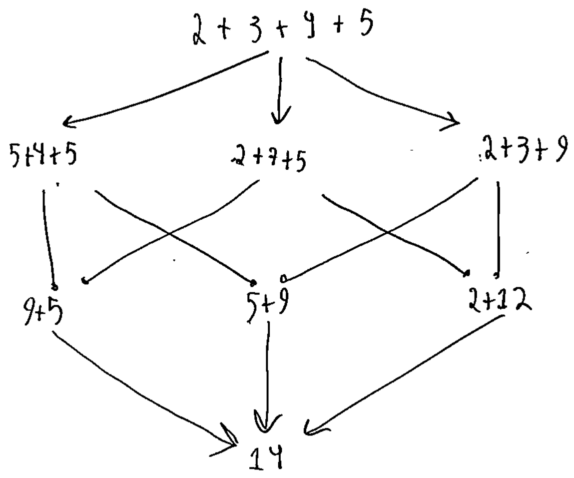
$$\underbrace{(10.70)}$$

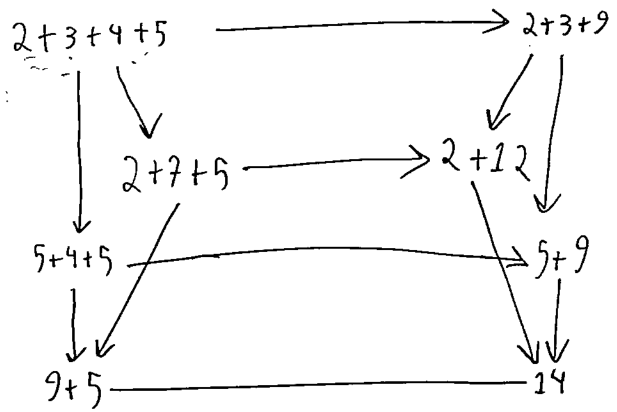
$$700$$

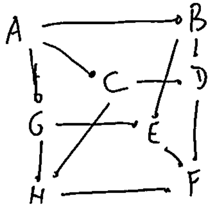
LA 29/AGO/2017











$\boxed{\text{LA } 29/\text{AGO}/2017}$

PRINCÍPIO:

TODAS AS SEQUÊNCIAS DE REDUÇÃO "CONVERGEM".

$(a+b)(a-b) \to a^2-b^2$

$(2+3)\cdot(4+5) \to$

45

ESSE PRINCÍPIO VAI VALER SEMPRE.

$\sum_{i=2}^{5} i^3 \to 2^3+3^3+4^3+5^3 \to \ldots$

$\sum_{i \text{ in } (2,3,4,5)} i^3$

IDEIA NOVA

$\sum_{i\in\{2,3,4,5\}} i^3$

$\sum_{i\in\{2,3,3,4,5\}} i^3$

$2^3+3^3+3^3+4^3+5^3 \to \ldots$

EXERCÍCIOS:

a) $(\lambda a.\ 10a)(2+3)$

b) $\underbrace{(\lambda a.10a)}_{\alpha}\underbrace{(\underbrace{(\lambda b.b+4)}_{\beta}(3))}_{\gamma}$

c) $\underbrace{(\underbrace{(\lambda a.\ (\underbrace{\lambda b.\ 10a+b}_{\beta}))}_{\alpha}(3))(4)}_{\gamma}$

d) $\underbrace{(\underbrace{(\lambda f.\ \underbrace{(\lambda a.\ f(f(a)))}_{\alpha})}_{\beta}\underbrace{(\lambda x.\ 10x)}_{\gamma})(7)}_{\delta}$

C) $((\lambda a.\ (\lambda b.\ 10a+b))(3))(4)$

$\underbrace{(\underbrace{(\lambda a.\ (\underbrace{\lambda b.\ 10a+b}_{\beta}))}_{\alpha}(3))(4)}_{\gamma}$

$\gamma(4) \to (\alpha(3))(4)$

$$
\begin{aligned}
\gamma(4) &= (\alpha(3))(4)\\
&= ((\lambda a.\ \beta)(3))(4)\\
&= (\beta[a:=3])(4)\\
&= ((\lambda b.\ 10a+b)[a:=3])(4)\\
&= ((\lambda b.\ 10\cdot 3+b))(4)\\
&= (\lambda b.\ 30+b)(4)\\
&\overset{\text{ok}}{=} (30+b)[b:=4]\\
&\overset{\text{ok}}{=} (30+4)\\
&\overset{\text{ok}}{=} (34)
\end{aligned}
$$

LA 5/set/2017

$A = \{1,2\}$

$B = \{3,4\}$

$C = \{30,40\}$

$D = \{10,20\}$

$f = \{(3,30),(4,40)\}$

$g = \{(1,10),(2,20)\}$

a) $A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

b) $A \times D = \{(1,10),(1,20),(2,10),(2,20)\}$

$A \to D = \left\{ \begin{Bmatrix}(1,10)\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,10)\\(2,20)\end{Bmatrix}, \begin{Bmatrix}(1,20)\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,20)\\(2,20)\end{Bmatrix} \right\}$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\mathrm{app}}{(\pi p, f(\pi' p))}\mathrm{pair} \qquad \dfrac{\dfrac{(1,3)}{1}\pi \qquad \dfrac{\dfrac{(1,3)}{3}\pi' \quad f}{30}\mathrm{app}}{(1,30)}\mathrm{pair}$$

$$\dfrac{\dfrac{\dfrac{p}{\pi p}\pi \quad g}{g(\pi p)} \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}}{(g(\pi p), f(\pi' p))}\mathrm{pair}$$

c)

$p = (1,3)$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{(\pi' p)}\pi' \quad f}{f(\pi' p)}\mathrm{app}}{(\pi p, f(\pi' p))}\mathrm{pair}$$

$(1,30)$

$p = (1,4)$

$$\dfrac{\dfrac{1,4}{1}\pi \qquad \dfrac{\dfrac{1,4}{4}\pi' \quad f}{40}\mathrm{app}}{(1,40)}\mathrm{pair}$$

$p = (2,3)$

$$\dfrac{\dfrac{(2,3)}{2}\pi \qquad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\mathrm{app}}{(2,30)}\mathrm{pair}$$

$p = (2,4)$

$$\dfrac{\dfrac{(2,4)}{2}\pi \qquad \dfrac{\dfrac{(2,4)}{4}\pi' \quad f}{40}\mathrm{app}}{(2,40)}\mathrm{pair}$$

d) $(\lambda p : A \times B . (\pi p, f(\pi' p))) = \{((1,3),(1,30)), ((1,4),(1,40)), ((2,3),(2,30)), ((2,4),(2,40))\}$

IN OUT

$\lambda p : A \times B$

e) Yes!

$A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

$A \to D = \left\{ \begin{Bmatrix}(1,10),\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,10),\\(2,20)\end{Bmatrix}, \begin{Bmatrix}(1,20),\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,20),\\(2,20)\end{Bmatrix} \right\}$

$(\pi p, f(\pi' p)) =$

$$\dfrac{\dfrac{(1,3)}{1}\pi \qquad \dfrac{\dfrac{(1,3)}{3}\pi' \quad \{(3,30),(4,40)\}}{30}\mathrm{app}}{(1,30)}\mathrm{pair} \qquad \dfrac{\dfrac{(1,4)}{1}\pi \qquad \dfrac{\dfrac{(1,4)}{4}\pi' \quad f}{40}\mathrm{app}}{(1,40)}\mathrm{pair}$$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{\pi'}\pi' \quad f}{f(\pi' p)}\mathrm{app}}{(\pi p, f(\pi' p))}\mathrm{pair}$$

$$\dfrac{\dfrac{(2,3)}{2}\pi \qquad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\mathrm{app}}{(2,30)}\mathrm{pair} \qquad \dfrac{\dfrac{(2,4)}{2}\pi \qquad \dfrac{\dfrac{(2,4)}{4}\pi' \quad f}{40}\mathrm{app}}{(2,40)}\mathrm{pair}$$

LA 5/set/2017

$A = \{1,2\}$

$B = \{3,4\}$

$C = \{30,40\}$

$D = \{10,20\}$

$f = \{(3,30),(4,40)\}$

$g = \{(1,10),(2,20)\}$

$D \times C = \{(10,30),(10,40),(20,30),(20,40)\}$

a) $A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

b) $A \times D = \{(1,10),(1,20),(2,10),(2,20)\}$

$A \to D = \left\{ \begin{Bmatrix}(1,10)\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,10)\\(2,20)\end{Bmatrix}, \begin{Bmatrix}(1,20)\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,20)\\(2,20)\end{Bmatrix} \right\}$

$A \times D \to D \times C = \{$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(\pi p, f(\pi' p))}\text{pair} \qquad \dfrac{\dfrac{(1,3)}{1}\pi \qquad \dfrac{\dfrac{(1,3)}{3}\pi' \quad f}{30}\text{app}}{(1,30)}\text{pair}$$

F

$$\dfrac{\dfrac{\dfrac{p}{\pi p}\pi \quad g}{g(\pi p)}\text{app} \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(g(\pi p), f(\pi' p))}\text{pair}$$

$(10,30)$

$p = (2,3)$

$$\dfrac{\dfrac{\dfrac{(2,3)}{2}\pi \quad g}{20}\text{app} \qquad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\text{app}}{(20,30)}\text{pair}$$

$p=(2,3)$

$$\dfrac{\dfrac{(2,3)}{2}\pi \qquad \dfrac{\dfrac{(2,3)}{3}\pi' \quad f}{30}\text{app}}{(2,30)}\text{pair}$$

1,4

10,

$p=(2,4)$

$$\dfrac{\dfrac{(2,4)}{2}\pi \qquad \dfrac{\dfrac{(2,4)}{4}\pi' \quad f}{40}\text{app}}{(2,40)}\text{pair}$$

$$\dfrac{\dfrac{\dfrac{p}{\pi p}\pi \quad g}{g(\pi p)}\text{app} \qquad \dfrac{\dfrac{p}{\pi' p}\pi' \quad f}{f(\pi' p)}\text{app}}{(g(\pi p), f(\pi' p))}\text{pair}$$

$(20,30)$

$A \times B = \{(1,3),(1,4),(2,3),(2,4)\}$

$A \to D = \left\{ \begin{Bmatrix}(1,10),\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,10),\\(2,20)\end{Bmatrix}, \begin{Bmatrix}(1,20),\\(2,10)\end{Bmatrix}, \begin{Bmatrix}(1,20),\\(2,20)\end{Bmatrix} \right\}$

$(\pi p, f(\pi' p)) =$

$\lambda p : A \times B . (\pi p, f(\pi' p)) =$

$\{((1,3),(1,30)),$
$((1,4),(1,40)),$
$((2,3),(2,30)),$
$((2,4),(2,40))\}$

$$\dfrac{\dfrac{p}{\pi p}\pi \qquad \dfrac{\dfrac{p}{\pi'}\pi' \quad f}{f(\pi' p)}\text{app}}{(\pi p, f(\pi' p))}\text{pair}$$

H) $(\lambda p : A \times B . (g(\pi p), f(\pi' p)))$

$((1,3),(10,30)),$
$((1,4),(10,40)),$
$((2,3),(20,30)),$
$((2,4),(20,40))$

False

D) $(\lambda p : A \times B . (\pi p, f(\pi' p)))$ - $\{$ in out
$((1,3),(1,30)),$
$((1,4),(1,40)),$
$((2,3),(2,30)),$
$((2,4),(2,40))\}$

$\lambda p : A \times B$

C) Yes!

$\boxed{\text{LA 5/SET/2017}}$

$f : A \to B$
$g : B \times C \to D$
$h : B \to (C \to D)$
$k : D \to E$

ⓐ $\underbrace{\lambda \underbrace{p}_{:A\times C} : A\times C \,.\, \underbrace{(\underbrace{f(\underbrace{\pi p}_{:A})}_{:B},\ \underbrace{\pi' p}_{:C})}_{:B\times C}}_{A\times C \to B\times C}$

ⓑ $\underbrace{\lambda \underbrace{q}_{:B\times C} : B\times C \,.\, \underbrace{(\underbrace{h(\underbrace{\pi q}_{:B})}_{:C\to D})(\underbrace{\pi' q}_{:C})}_{:D}}_{:B\times C \to D}$

ⓒ $\underbrace{\lambda \underbrace{b}_{:B} : B \,.\, \underbrace{\lambda \underbrace{c}_{:C} : C \,.\, \underbrace{g\underbrace{(\underbrace{b}_{:B},\underbrace{c}_{:C})}_{:B\times C}}_{:D}}_{:C\to D}}_{:B \to (C\to D)}$

ⓓ $\underbrace{\lambda \underbrace{\varphi}_{:C\to D} : C\to D \,.\, \lambda \underbrace{c}_{:C} : C \,.\, \underbrace{k\,\underbrace{(\underbrace{\underbrace{\varphi}_{:C\to D}\, \underbrace{c}_{:c}}_{:D})}_{:D}}_{:E}}_{:C\times E}$

$:(C\to D) \to C\times E$

$\varphi \quad \infty$

$2 \in \{2,3,4\}$
$2 \in \{1,2,3\}$

$\dfrac{p : A\times C}{\pi p : A}$

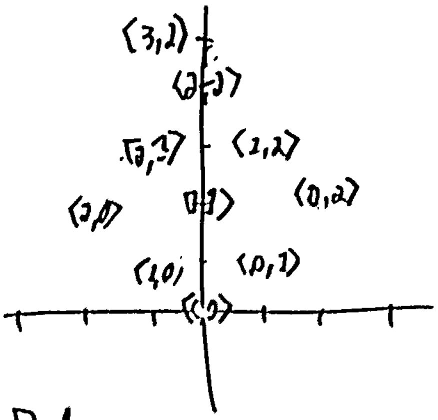
a) $(\times c) f := \underbrace{\lambda \underbrace{p}_{A\times C} : A\times C \,.\, \underbrace{(\underbrace{\underbrace{f}_{A\to B}(\underbrace{\pi}_{}\underbrace{p}_{:A\times C})}_{:B},\ \underbrace{\pi' p}_{:C})}_{B\times C}}_{:A\times C \to B\times C}$

b) $h^\flat := \underbrace{\lambda q : B\times C \,.\, \underbrace{(\underbrace{\underbrace{h}_{:B\to(C\to D)}(\pi \underbrace{q}_{:B\times C})}_{:C\to D})(\pi' \underbrace{q}_{:B\times C})}_{:D}}_{:B\times C \to D}$

c) $g^\# := \underbrace{\lambda \underbrace{b}_{:B} : B \,.\, \underbrace{\lambda \underbrace{c}_{:C} : C \,.\, \underbrace{\underbrace{g}_{:B\times C\to D}\underbrace{(\underbrace{b}_{:B}, \underbrace{c}_{:C})}_{:B\times C}}_{:D}}_{:C\to D}}_{:B \to (C\to D)}$

$\boxed{\text{LA 12/SET/2017}}$

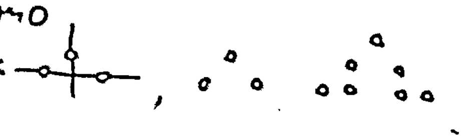
HOJE: VAMOS INTERROMPER
A PROGRAMAÇÃO NORMAL
(λ-CÁLCULO TIPADO) PRA
COMEÇAR A VER A
RELAÇÃO ENTRE λ-CÁLCULO
E LÓGICA!



a) $\lambda \ell_n : B.\ell$

PLANAR HEYTING
ALGEBRAS FOR CHILDREN
→ FIRST PAPER
→ SEÇÕES:

1. POSITIONAL NOTATIONS

ESCREVA COMO
CONJUNTO: 

2. ZDAGs

SEJA H = .
REPRESENTE (H, BPM(H))
COMO GRAFO DIRECIONADO
E COMO PAR DE
CONJUNTOS.

3: LR-COORDINATES

PARA CADA UM DESTES PONTOS
DÊ AS COORDENADAS (x, y) DELE
E REPRESENTE-OS EM $\mathbb{Z}_2$.

$\langle 0,0\rangle, \langle 1,0\rangle, \langle 2,0\rangle, \langle 0,1\rangle, \langle 0,3\rangle, \langle 1,3\rangle, \langle -1,0\rangle, \langle 0,-1\rangle, \langle -1,-1\rangle.$

EXERCÍCIO (GRANDE):

SEJA B =

```
       32
         22
      21    12
   20    11    02
      10    01
         00
```

CALCULE, E REPRESENTE EM NOTAÇÃO POSICIONAL QUANDO POSSÍVEL:

a) $\lambda lr{:}B.\ l$
b) $\lambda lr{:}B.\ r$
c) $\lambda lr{:}B.\ (l \leq 1)$
d) $\lambda lr{:}B.\ (r \geq 1)$
e) $\lambda lr{:}B.\ lr \leq 11$
f) $\lambda lr{:}B.\ lr \& 12$
g) $\lambda lr{:}B.\ \mathrm{valid}(\langle l+1, r\rangle)$
h) $\lambda lr{:}B.\ lr$ leftof 11
i) $\lambda lr{:}B.\ lr$ leftof 12
j) $\lambda lr{:}B.\ lr$ above 11
k) $\lambda lr{:}B.\ \mathrm{ne}(lr)$
l) $\lambda lr{:}B.\ \mathrm{nw}(lr)$
m) $20 \to 11$
n) $02 \to 11$
o) $22 \to 11$
p) $00 \to 11$
q) $\lambda lr{:}B.\ \neg lr$
r) $\lambda lr{:}B.\ \neg\neg lr$
s) $\lambda lr{:}B.\ (lr = \neg\neg lr)$

ⓐ $\lambda lr : B.\ l$

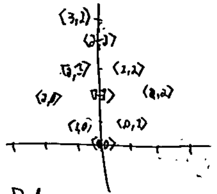
```
       3
         2
      2     1
   2     1     0
      1     0
         0
```

lr	l	r	(l≤1)	(r≥1)	(lr≤11)
00	0	0	1	0	1
01	0	1	1	0	1
02	0	2	1	1	0
10	1	0	1	0	1
11	1	1	1	0	1
12	1	2	1	1	0
20	2	0	0	0	0
21	2	1	0	0	0
22	2	2	0	1	0
32	3	2	0	1	0

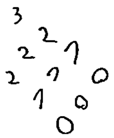
LA 12/SET/2017

lr	l	r	$l \leq 1$	$r \geq 1$	$lr \leq 11$
00	0	0	1	0	1
10	1	0	1	0	1
01	0	1	1	1	1
20	2	0	0	0	0
11	1	1	1	1	1
02	0	2	1	1	0
21	2	1	0	1	0
12	1	2	1	1	0
22	2	2	0	1	0
32	3	2	0	1	0

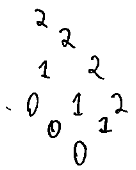
lr & 12	valid($\langle l+1, r\rangle$)	leftof 11
		10
00	1	10
10	1	11
01	1	20
20	0	20
11	1	11
12	1	11
12	0	21
12	1	11
	1	21
	0	31



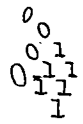
a) $\lambda lr : B . l$



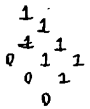
b)



c)

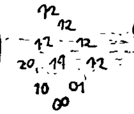


d)

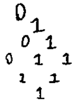


E)

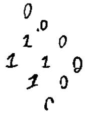
F)



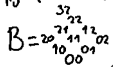
G)



h)



Exercício (grande):

Seja B = 

Calcule, e represente em notação posicional quando possível:

a) $\lambda lr:B.\ l$
b) $\lambda lr:B.\ r$
c) $\lambda lr:B.\ (l \leq 1)$
d) $\lambda lr:B.\ (r \geq 1)$
e) $\lambda lr:B.\ lr \leq 11$
f) $\lambda lr:B.\ lr \,\&\, 12$
g) $\lambda lr:B.\ \mathrm{valid}(\langle l+1, r\rangle)$
h) $\lambda lr:B.\ lr$ leftof 11
i) $\lambda lr:B.\ lr$ leftof 12
j) $\lambda lr:B.\ lr$ above 11
k) $\lambda lr:B.\ \mathrm{ne}(lr)$
l) $\lambda lr:B.\ \mathrm{nw}(lr)$
m) $20 \to 11$
n) $02 \to 11$
o) $22 \to 11$
p) $00 \to 11$
q) $\lambda lr:B.\ \neg lr$
r) $\lambda lr:B.\ \neg\neg lr$
s) $\lambda lr:B.\ (lr = \neg\neg lr)$

00 · 12

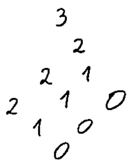
& δ 00

&

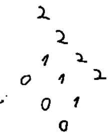
lr	l	r	lr leftof 11	valid($\langle l+1, r\rangle$)
00	0	0	0	1
01	0	1	0	1
02	0	2	0	1
10	1	0	1	1
11	1	1	1	1
12	1	2	0	1
20	2	0	1	0
21	2	1	1	0
22	2	2	0	1
32	3	2	0	0

$\Omega = B$

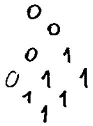
a) $\lambda lr : B.\ l$



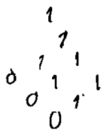
b) $\lambda lr : B.\ r$



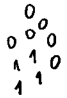
c) $\lambda lr : B.\ (l \leq 1)$



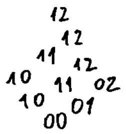
d) $\lambda lr : B\ (r \geq 1)$



e) $\lambda lr : B.\ lr \leq 11$

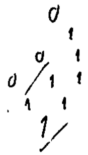


f) $\lambda lr : B.\ lr \,\&\, 12$

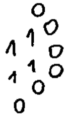


$l \geq l'$
&
$r \leq r'$

g) $\lambda lr : B.\ \mathrm{valid}(\langle l+1, r\rangle)$



h) $\lambda lr : B.\ lr$ leftof 11

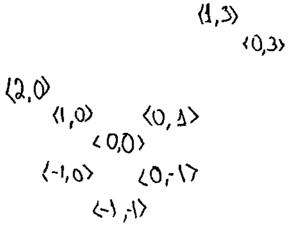


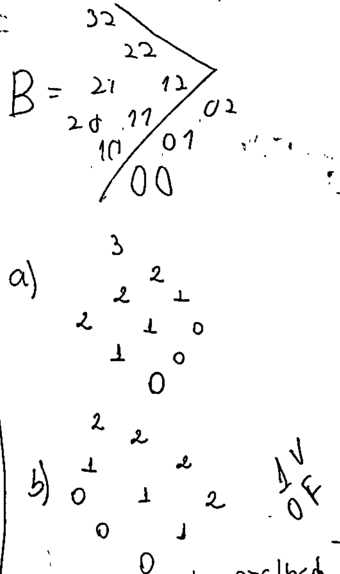
LA 26/set/2017

1) Positional notations
Escreva como conjunto:

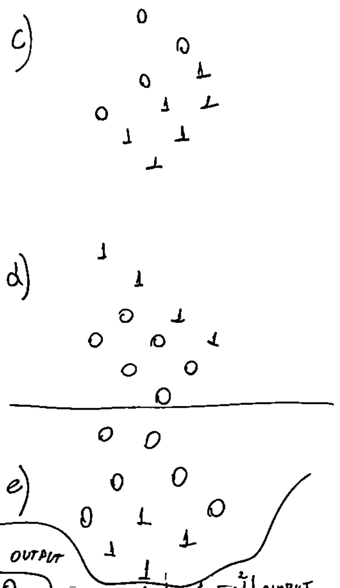
2) ZDAGs
Seja H = 
Represente (H, BPM(H)) como grafo direcionado e como par de conjuntos.

3) LR-coordinates
Para cada um destes pontos dê as coordenadas (x,y) dele e represente-o em $\mathbb{Z}^2$.
$\langle 0,0\rangle, \langle 1,0\rangle, \langle 2,0\rangle, \langle 0,1\rangle, \langle 0,3\rangle, \langle 1,3\rangle, \langle -1,0\rangle, \langle 0,-1\rangle, \langle -1,-1\rangle$.

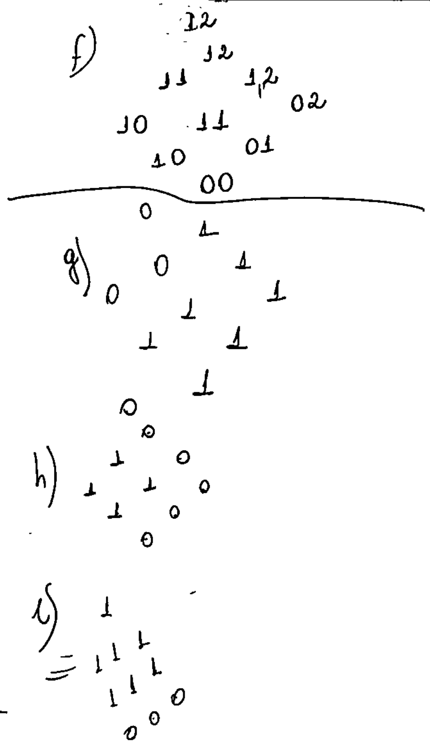




ab	$a \geq c_1$	$b \leq d_1$	OUTPUT
00	0	1	0
01	0	1	0
01	0	0	0
10	1	1	1
11	1	1	1
12	1	0	0
20	1	1	1
21	1	1	1
22	1	0	0
32	1	0	0

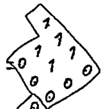


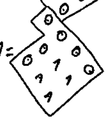
ab	$a \geq \overset{1}{c}$	$b \leq \overset{2}{d}$	OUTPUT
00	0	1	0
01	0	1	0
02	0	1	0
10	1	1	1
11	1	1	1
12	1	1	1
20	1	1	1
21	1	1	1
22	1	1	1
3,2	1	1	1



(a,b) leftof (c,d)

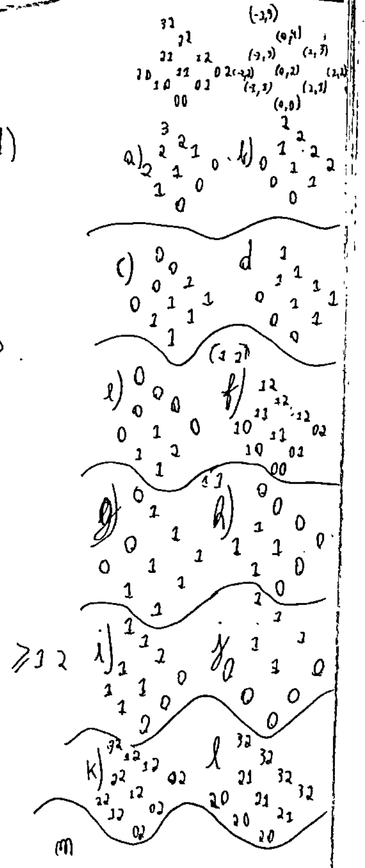
$a \geq c \land b \leq d$

λ lr:B. lr above 11 = 

λ lr:B. lr below 11 = 

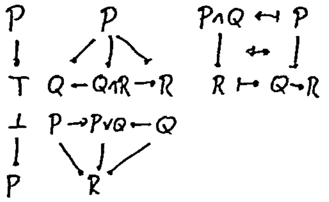
$(0, 0+1)$

ne(00) = ne(01) = ne(02) 02
ne(01) = 02
ne(02) = 02
ne(10) = 12



LA 10/OUT/2017

ÁLGEBRAS DE HEYTING
OBEDECEM ISSO AQUI:



$(P \le Q) \wedge (P \le R) \leftrightarrow (P \le \underbrace{(Q \wedge R)}_{?})$ ①

$\lambda P.(P \le Q \wedge P \le R) = \lambda P.\ P \le Q \wedge R$ ②

$(P \wedge Q \le R) \leftrightarrow (P \le Q \to R)$ ③

$\lambda P.(P \wedge Q \le R) = \lambda P.(P \le \underbrace{(Q \to R)}_{?})$ ④

SEJA Ω =

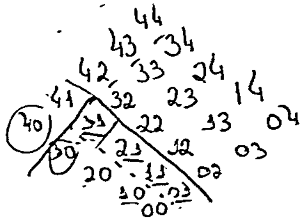
SE Q=31 E R=12,
ENCONTRE O ÚNICO VALOR PRO "?"
QUE OBEDECE ISTO:

$\lambda P.(P \wedge Q \le R) = \lambda P.(P \le ?)$ $? = 14$

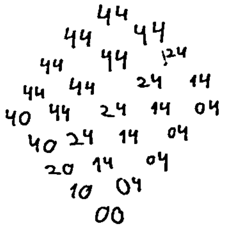
$\lambda P.((P \wedge \overset{31}{Q}) \le \overset{12}{R})$

$\langle \min(a,3), \min(b,1) \rangle$

$\lambda P.(P \wedge Q)$



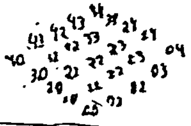
R	$\overset{31}{Q} \to R$
00	00
01	04
02	04
03	04
04	04
10	10
11	14
12	14
13	14
14	14
20	20
21	24
22	24
23	24
24	24
30	40
31	44
32	44
33	44
34	44
40	40
41	44
42	44
43	44
44	44



R	$\overset{31}{Q} \to R$
00	00
01	04
02	04
03	04
04	04
10	10
11	14
12	14
13	14
14	14
20	20
21	24
22	24
23	24
24	24
30	40
31	44
32	44
33	44
34	44
40	40
41	44
42	44
43	44
44	

Q ↓R T → sup(Ω)
← me
→ mw
1 (c,d)

Q=31 R=12



$(\lambda P.\ P \wedge Q) =$

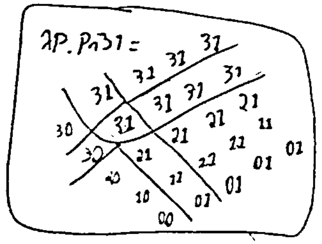
$(\lambda P.\ P \wedge 31) =$

$= (\lambda P.(P \wedge \overset{Q}{31}) \le \overset{R}{12})$

$= (\lambda P.(P \le ?))$ $? = 14$

Q → R

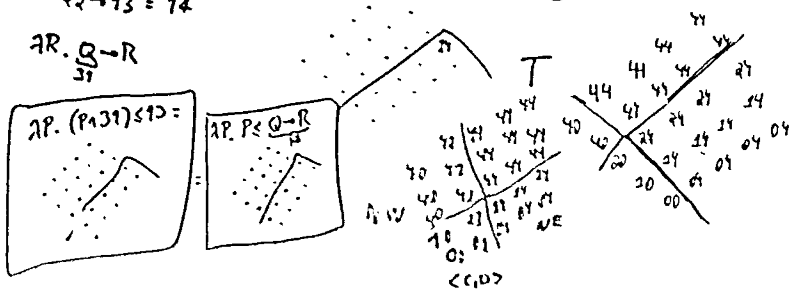
22 → R = ?



Q → R
22 → 12 = 14
22 → 13 = 14

$\lambda R.\ \underset{31}{Q} \to R$

B



LA 10/out/2017

ÁLGEBRAS DE HEYTING

OBEDECEM ISSO AQUI:

$$\begin{tikzcd} P \arrow[d] \\ \top \end{tikzcd} \qquad \begin{tikzcd} & P \arrow[dl] \arrow[d] \arrow[dr] & \\ Q & Q \land R \arrow[l] \arrow[r] & R \end{tikzcd} \qquad \begin{tikzcd} P \land Q \arrow[d] \arrow[r, phantom, "\leftarrow\!\mapsto"] & P \arrow[d] \\ R & Q \to R \arrow[l, phantom, "\mapsto"] \end{tikzcd}$$

$$\begin{tikzcd} \bot \arrow[d] \\ P \end{tikzcd} \qquad \begin{tikzcd} P \arrow[r] \arrow[dr] & P \lor Q \arrow[d] & Q \arrow[l] \arrow[dl] \\ & R & \end{tikzcd}$$

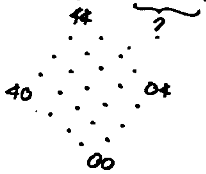
$(P \leq Q) \land (P \leq R) \leftrightarrow (P \leq \underbrace{(Q \land R)}_{?}))$ ①

$\lambda P.(P \leq Q \land P \leq R) = \lambda P.\ P \leq \overset{?}{Q \land R}$ ②

$(P \land Q \leq R) \leftrightarrow (P \leq Q \to R)$ ③

$\lambda P.(P \land Q \leq R) = \lambda P.(P \leq \underbrace{(Q \overset{B}{\to} R)}_{?})$ ④

SEJA $\Omega =$



SE $Q = 31$ E $R = 12$,
ENCONTRE O ÚNICO VALOR PRO "?" QUE OBEDECE ISTO:

$\lambda P.(\underbrace{P \land Q}\leq R) = \lambda P.(P \leq ?)$

(1→) $P \leq Q \land P \leq R \to P \leq Q \land R$

(1←) $P \leq Q \land P \leq R \leftarrow P \leq Q \land R$

(1←a) $P \leq Q \qquad \leftarrow P \leq Q \land R$

(1←b) $\qquad P \leq R \leftarrow P \leq Q \land R$

(3→) $P \land Q \leq R \to P \leq Q \to R$

(3←) $P \land Q \leq R \leftarrow P \leq Q \to R$

(1→): $\dfrac{P \leq Q \quad P \leq R}{P \leq Q \land R}$

(1←a): $\dfrac{P \leq Q \land R}{P \leq Q}$

(1←b): $\dfrac{P \leq Q \land R}{P \leq R}$

R	$\overset{31}{\widetilde{Q}} \to R$
00	00
01	04
02	04
03	04
04	04
10	10
11	14
12	14
13	14
14	14
20	20
21	24
22	24
23	24
24	
30	40
31	44
32	44
33	44
34	44
40	40
41	44
42	44
43	44
44	44

LA 17/out/2017

HOJE: TOPOLOGIAS!

ESTAMOS NO AUDITÓRIO ...

IRENE

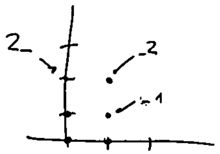
(25)
14

5
2
1
1 4

3 4
2

$X\_ = (0\ X)$

$\_X = (1\ X)$



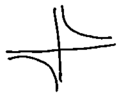
$(-\infty, 0]$ $[1, \infty)$

$(0, 1)$

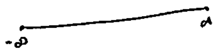
$(0, \infty)$

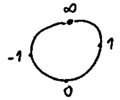
$\mathbb{R} = (-\infty, \infty)$

$\mathbb{R} \cup \{-\infty, +\infty\} = [-\infty, +\infty]$

$1/x =$ 

$^{1}{}_{0}{}^{0} = (-\infty, 0]$





$(-\infty, 42]$ $(42, 99)$ $[99, +\infty)$

$(-\infty, \infty)$

$\emptyset$

$0_0{}^0 \Rightarrow \emptyset$

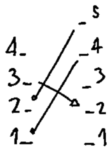
$0_1{}^0 \Rightarrow (0,1)$

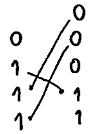
$0_1{}^1 \Rightarrow (0,1) \cup [1,\infty) = (0,\infty)$

$1_1{}^0 \Rightarrow (-\infty,0] \cup [0,1) = (-\infty,1)$

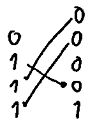
$= (-\infty, \infty)$

$1_1{}^1 \Rightarrow$

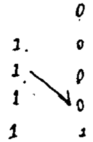




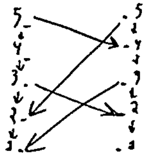
pile(32)

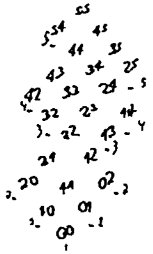


pile(31)



5_ = 54
4_ = 42
3_ = 32
2_ = 20
1_ = 10





5_ = 54 GAP: 5_ → _4
4_ = 42
3_ = 32 GAP: 3_ → _2
2_ = 20
1_ = 10

5 = 21 GAP: 2 ← _5
_4 = 14
3 = 13 GAP: 1 ← _3
_2 = 02
_1 = 01

LA 31/OUT/2017

$(2,4) \cap (3,5) = (3,4)$

$(2,4) \cap [3,5] = [3,4)$

$(2,4) \cup (3,5) = (2,5)$

$(2,4) \cup [3,5] = (2,5]$

$(\frac{1}{2},1) \cup (\frac{1}{3},1) \cup (\frac{1}{4},1) \cup \ldots =$

$= (0,1)$

$[\frac{1}{2},1] \cup [\frac{1}{3},1] \cup [\frac{1}{4},1] \cup \ldots = \quad (0,1]$

$(0,2) \cup (1,3) \cup (2,4) \cup (3,5) \cup \ldots = ?$

$(0,+\infty)$

$(-1,\frac{1}{2}) \cap (-1,\frac{1}{3}) \cap (-1,\frac{1}{4}) \cap \ldots = (-1,0]$

$(0,\frac{1}{2}) \cap (0,\frac{1}{3}) \cap (0,\frac{1}{4}) \cap \ldots = \emptyset$

$\mathbb{R} \setminus (2,3) = (-\infty,2] \cup [3,+\infty)$

$\mathbb{R} \setminus [2,3] = (-\infty,2) \cup (3,+\infty)$

UM CONJUNTO $A \subset \mathbb{R}$
É FECHADO SE E SÓ SE
$\mathbb{R} \setminus A$ É ABERTO.

$\emptyset$ É ABERTO

$\{1,2,3,4\} \setminus \{3,4,5,6\} = \{1,2\}$

SEJA $V = \therefore$

ENTÃO $\mathcal{P}(V) = \left\{ \substack{00\\0}, \substack{00\\1}, \substack{01\\0}, \substack{01\\1}, \substack{10\\0}, \substack{10\\1}, \substack{11\\0}, \substack{11\\1} \right\}$

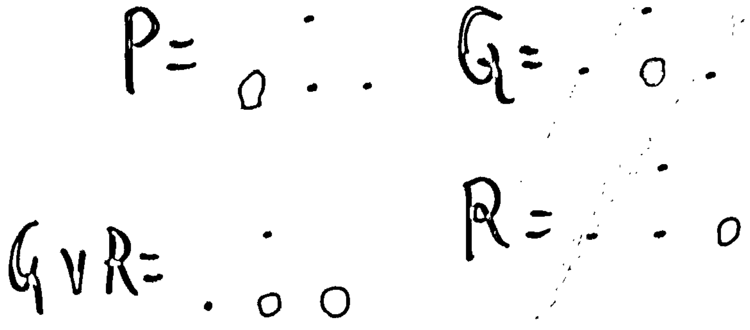
$\mathcal{O}(V) = \left\{ \substack{00\\0}, \substack{00\\1}, \substack{01\\1}, \substack{10\\1}, \substack{11\\1} \right\}$

$(V, \mathcal{P}(V))$,

$(V, \mathcal{O}(V))$ E

$\left(V, \left\{ \substack{00\\0}, \substack{11\\1} \right\}\right)$ SÃO ESPAÇOS TOPOLÓGICOS.

$P =$ $\quad Q =$ $\quad R =$ $\quad Q \vee R =$



$P \to (Q \vee R)$

$\mathrm{int}\ P \xrightarrow{M} (Q \vee R)$

P: 0 / 100

Q: 0 / 010

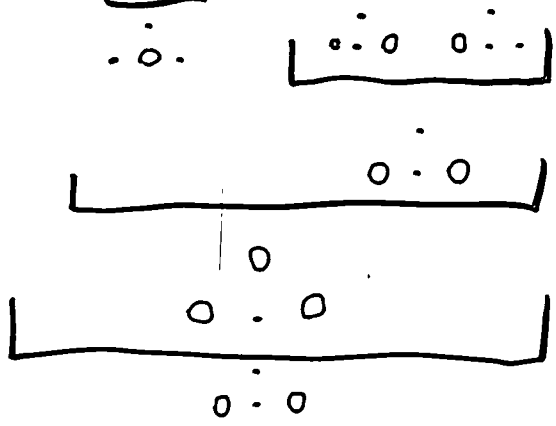
R: 0 / 001

$Q \vee R$: 0 / 011

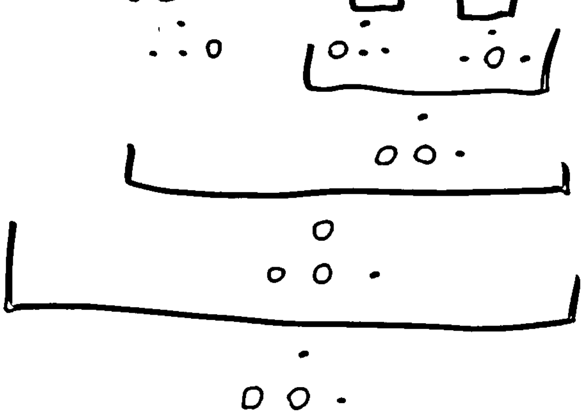
$P \xrightarrow{M} (Q \vee R)$: 1 / 011

$\mathrm{int}\ P \xrightarrow{M} (Q \vee R)$: 0 / 011

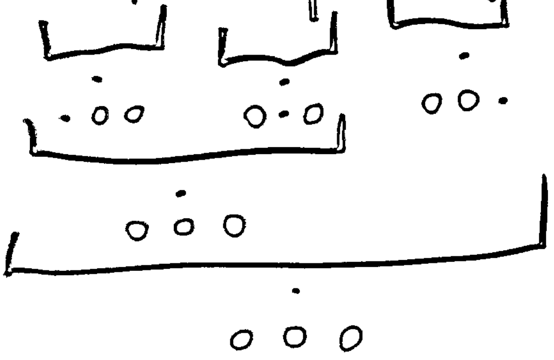
$\mathrm{int}(Q \xrightarrow{M} (R \vee P))$



$\mathrm{int}(R \xrightarrow{M} (P \vee Q))$



$S_P \vee S_Q \vee S_R$



$\boxed{\text{LA 7/NOV/2017}}$

PRÓXIMOS ASSUNTOS
DO CURSO:
CATEGORIAS,
COMO "OMITIR VARIÁVEIS",
LINGUAGEM DE
PROGRAMAÇÃO COM "λ".

$$\dfrac{\dfrac{\dfrac{p:A\times C}{\pi p:A}\pi \quad f:A\to C}{f(\pi p):B}\text{APP} \quad \dfrac{p:A\times C}{\pi' p:C}\pi'}{\dfrac{(f(\pi p),\pi' p):B\times C}{\lambda p:A\times C.(f(\pi p),\pi' p):A\times C\to B\times C}\lambda}\text{PAIR}$$

p.13:

$$\dfrac{f,p\vdash(\pi p,f(\pi' p))}{f\vdash\lambda p:A\times B.(\pi p,f(\pi' p))}\lambda$$

$$\dfrac{f,p\vdash(\pi p,f(\pi' p))}{p\vdash\lambda f:B\to C.(\pi p,f(\pi' p))}\lambda$$

$$\dfrac{f,p\vdash(\pi p,f(\pi' p))}{f,p\vdash\lambda d:D.(\pi p,f(\pi' p))}\lambda$$

$$\dfrac{\dfrac{\dfrac{q:B\times C}{\pi' q:C}\pi' \quad \dfrac{\dfrac{q:B\times C}{\pi q:B}\pi \quad h:B\to(C\to D)}{h(\pi q):C\to D}\text{APP}}{h(\pi q)(\pi' q):D}\text{APP}}{\lambda q:B\times C.h(\pi q)(\pi' q):B\times C\to D}\lambda$$

$$\dfrac{\dfrac{\dfrac{Q\land R}{R} \quad \dfrac{\dfrac{Q\land R}{Q} \quad Q\to(R\to S)}{R\to S}}{S}}{Q\land R\to S}$$

(A, B, a, p), ONDE:
A é um conjunto,
B " " "
$a\in A$,
$p\in A\times B$.

3 - 4
-4

$\mathbb{N}$
A
LIST(A)
TREE(A)

$$\dfrac{p:P \quad f:P\to Q}{fp:Q}$$

23 22
21 12
20 11 02
10 01
00

$P\to Q$

P

$A\to B$

$$\dfrac{P\land Q \quad Q\to R}{P\land R}$$

$$\dfrac{P \quad Q}{\qquad}\ \Big|\ \dfrac{G\to R}{\qquad}$$

$$\dfrac{P \quad G}{P\,\&\,G}\&I \qquad \dfrac{P\,\&\,G}{P}\&E_1 \qquad \dfrac{P\,\&\,G}{G}\&E_2$$

$$\dfrac{\begin{matrix}P \quad [G]^1\\ \vdots\\ R\end{matrix}}{G\to R}\to I$$

$\boxed{\text{LA 21/NOV/2017}}$

HOJE:
PRODUTOS ETC
NO LIVRO DO AWODEY!
DUALIDADE!

NA AULA PASSADA:

$\{3,4\} \xleftarrow{\pi_1} \left\{\begin{matrix}(3,5), (3,6),\\ (4,5), (4,6)\end{matrix}\right\} \xrightarrow{\pi_2} \{5,6\}$

$\{3,4\} \xleftarrow{\pi_1} \left\{\begin{matrix}35, 36\\ 45, 46\end{matrix}\right\} \xrightarrow{\pi_2} \{5,6\}$

SÃO "PRODUCT DIAGRAMS", MAS ISTO AQUI NÃO É:

$\{3,4\} \xleftarrow{\pi_1} \left\{\begin{matrix}35,\\ 46, 64\end{matrix}\right\} \xrightarrow{\pi_2} \{5,6\}$

$$\begin{tikzcd}
 & \{0\} \arrow[dl, "0\mapsto 3"'] \arrow[d, "0\mapsto 35"] \arrow[dr, "0\mapsto 5"] & \\
\{3,4\} & \left\{\begin{matrix}35,\\ 46,64\end{matrix}\right\} \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] & \{5,6\}
\end{tikzcd}$$

$$\begin{tikzcd}
 & \{0\} \arrow[dl, "0\mapsto 3"'] \arrow[d, "☹"] \arrow[dr, "0\mapsto 6"] & \\
\{3,4\} & \left\{\begin{matrix}35,\\ 46,64\end{matrix}\right\} \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] & \{5,6\}
\end{tikzcd}$$

$$\begin{tikzcd}
 & \{0\} \arrow[dl, "0\mapsto 4"'] \arrow[d, shift left, "0\mapsto 46"] \arrow[d, shift right, "0\mapsto 64"'] \arrow[dr, "0\mapsto 6"] & \\
\{3,4\} & \left\{\begin{matrix}35,\\ 46,64\end{matrix}\right\} \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] & \{5,6\}
\end{tikzcd}$$

$35\mapsto 3$, $46\mapsto 4$, $64\mapsto 4$ $35\mapsto 5$, $46\mapsto 6$, $64\mapsto 6$

$B \xleftarrow{\pi_1} \overset{P}{B\times C} \xrightarrow{\pi_2} C$ É UM "PRODUCT DIAGRAM"
QUANDO $\forall A.\ \forall A\xrightarrow{f}B,\ A\xrightarrow{g}C.\ \exists! h.\ (f=h;\pi_1 \land g=h;\pi_2)$

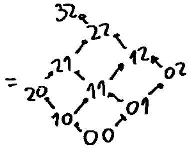
$A\xrightarrow{h}P$ $f=\pi_1\circ h \land g=\pi_2\circ h$

$$\begin{tikzcd}
 & A \arrow[dl, "f"'] \arrow[d, "h"] \arrow[dr, "g"] & \\
B & \underset{P}{B\times C} \arrow[l, "\pi_1"] \arrow[r, "\pi_2"'] & C
\end{tikzcd}$$

$$\begin{tikzcd}
 & \{0,1\} \arrow[dl, "\begin{matrix}0\mapsto 3\\1\mapsto 2\end{matrix}"'] \arrow[d, "\begin{matrix}0\mapsto 5?\\1\mapsto 4?\end{matrix}"] \arrow[dr, "\begin{matrix}0\mapsto 8\\1\mapsto 8\end{matrix}"] & \\
\{2,3\} & \left\{\begin{matrix}4, 5,\\ 6, 7\end{matrix}\right\} \arrow[l, "\begin{matrix}4\mapsto 2\\5\mapsto 3\\6\mapsto 2\\7\mapsto 3\end{matrix}"] \arrow[r, "\begin{matrix}4\mapsto 8\\5\mapsto 8\\6\mapsto 9\\7\mapsto 9\end{matrix}"'] & \{8,9\}
\end{tikzcd}$$

TODA ORDEM PARCIAL É UMA CATEGORIA.

$\mathrm{Hom}(A,B) = \begin{cases}\{(A,B)\} & \text{SE } A\le B,\\ \{\} & \text{SE NÃO.}\end{cases}$

$\mathbb{B} =$ 

$10 \le 12$? SIM
$21 \le 12$? NÃO
$22 \le 22$? SIM

$\mathrm{Hom}(10,12) = \{(10,12)\}$
$\mathrm{Hom}(21,12) = \{\}$
$\mathrm{Hom}(22,22) = \{(22,22)\}$

$$\begin{tikzcd}
 & 00 \arrow[dl, "(00,21)"'] \arrow[d, "(00,01)"] \arrow[dr, "(00,02)"] & \\
21 & 01 \arrow[l, "(01,21)"] \arrow[r, "(01,02)"'] & 02
\end{tikzcd}$$

É UM "PRODUCT DIAGRAM" EM $\mathbb{B}$.

$$\begin{tikzcd}
 & 11 \arrow[dl, "(11,21)"'] \arrow[d, "!!"] \arrow[dr, "!!"] & \\
21 & 01 \arrow[l, "(01,21)"] \arrow[r, "(01,02)"'] & 02
\end{tikzcd}$$

$$\begin{tikzcd}
 & A & \\
B \arrow[ur, "f"] \arrow[r, "i_1"'] & P \arrow[u] & C \arrow[ul, "g"'] \arrow[l, "i_2"]
\end{tikzcd}$$

$f=h\circ i_1 \land g=h\circ i_2$

$B \xrightarrow{i_1} P \xleftarrow{i_2} C$ É UM "COPRODUCT DIAGRAM" QUANDO ...

$\forall A, \forall B\xrightarrow{f}A, C\xrightarrow{g}A, \exists! P\xrightarrow{h}A \left(f=h\circ i_1 \land g=h\circ i_2\right)$

$$\begin{tikzcd}
 & A \arrow[dl, "f"'] \arrow[d, "h"] \arrow[dr, "g"] & \\
B & P \arrow[l, "i_1"] \arrow[r, "i_2"'] & C
\end{tikzcd}$$

é um produto.

$f=i_1\circ h \land g=i_2\circ h$

LA /NOV/2017

HOJE! DUALIDADE - MAS COMEÇANDO UM POUCO ADIANTE DE ONDE A GENTE CHEGOU NA AULA PASSADA...

① $B \xleftarrow{p_1} D \xrightarrow{p_2} C$ É UM "PRODUCT DIAGRAM" QUANDO

OBS: (B, C, D, p_1, p_2)

$\forall A.\ \forall f: A \to B.\ \forall g: A \to C.$
$\exists! h: A \to C.\ (p_1 \circ h = f \wedge p_2 \circ h = g)$

② $B \xrightarrow{p_1} D \xleftarrow{p_2} C$ É UM "COPRODUCT DIAGRAM" QUANDO

$\forall A.\ \forall f: B \to A.\ \forall g: C \to A.$
$\exists! h: C \to A.\ (h \circ p_1 = f \wedge h \circ p_2 = g)$

TEM UM OUTRO JEITO DE DEFINIR PRODUTOS E COPRODUTOS...

$$\begin{tikzcd} & A \arrow[dl, "f"'] \arrow[d, "h"] \arrow[dr, "g"] & \\ B & D \arrow[l, "p_1"] \arrow[r, "p_2"'] & C \end{tikzcd}$$

OLHE PRA CÁ:

$$w_A : \mathrm{Hom}(A, D) \to \mathrm{Hom}(A, B) \times \mathrm{Hom}(A, C)$$
$$h \mapsto (p_1 \circ h,\ p_2 \circ h)$$

TODO DIAGRAMA DESTA FORMA

$B \xleftarrow{\alpha} D \xrightarrow{\beta} C$

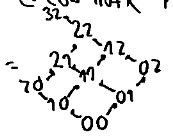
INDUZ UMA OPERAÇÃO COMO A "w_A" ACIMA PRA TODO A...

UM DIAGRAMA DESTA FORMA

$B \xleftarrow{p_1} D \xrightarrow{p_2} C$ É UM "PRODUCT DIAGRAM" QUANDO $\forall A$ A OPERAÇÃO w_A É UMA BIJEÇÃO...

OBS: SE w_A É UMA BIJEÇÃO ENTÃO $|\mathrm{Hom}(A,D)| = |\mathrm{Hom}(A,B) \times \mathrm{Hom}(A,C)|$
$= |\mathrm{Hom}(A,B)| \cdot |\mathrm{Hom}(A,C)|$ (★)

VAMOS USAR (★) PRA ENCONTRAR PRODUTOS AQUI...

$\mathbb{B} =$ 

DIGAMOS QUE $B = 21$ E $C = 02$.
DIGAMOS QUE $D = 00$.
ENCONTRE UM A TAL QUE
$|\mathrm{Hom}(A, \underbrace{D}_{00})| \neq |\mathrm{Hom}(A, \underbrace{B}_{21})| \cdot |\mathrm{Hom}(A, \underbrace{C}_{02})|$.

RESP: $A = 01$.

DIGAMOS QUE $B = 21$ E $C = 02$.
ENCONTRE UM D TAL QUE (★) VALHA PARA TODO A.

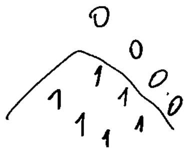
$|\mathrm{Hom}(A, \underbrace{B}_{21})| \cdot |\mathrm{Hom}(A, \underbrace{C}_{02})| = |\mathrm{Hom}(A, D)|$

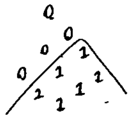
RESP: $D = 01$.

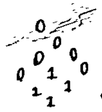
A	$\|\mathrm{Hom}(A,B)\|$	$\cdot\ \|\mathrm{Hom}(A,C)\|$	
00	1	1	1
01	1	1	1
02	0	1	0
10	1	0	0
11	1	0	0
12	0	0	0
20	1	0	0
21	1	0	0
22	0	0	0
32	0	0	0

USANDO A NOTAÇÃO POSICIONAL...

$\lambda A{:}\mathbb{B}.\ |\mathrm{Hom}(A,B)|$ $\quad$ $\lambda A{:}\mathbb{B}.\ |\mathrm{Hom}(A,C)|$







$\lambda A{:}\mathbb{B}.\ |\mathrm{Hom}(A,B)| \cdot |\mathrm{Hom}(A,C)|$

QUAL É O D TAL QUE $\lambda A{:}\mathbb{B}.\ |\mathrm{Hom}(A,D)|$ TEM O MESMO COMPORTAMENTO QUE ISTO?

$B = 21$ $\quad$ $C = 12$

COPRODUTOS (EM $\mathbb{B}$)

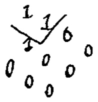
SE $B \xrightarrow{\alpha} D \xleftarrow{\beta} C$ É UM COPRODUCT DIAGRAM ENTÃO ESTA OPERAÇÃO

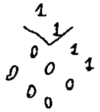
$$m_A : \mathrm{Hom}(D, A) \to \mathrm{Hom}(B, A) \times \mathrm{Hom}(C, A)$$
$$h \mapsto (h \circ p_1,\ h \circ p_2)$$

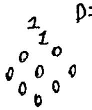
É UMA BIJEÇÃO PARA TODO A...

ISTO IMPLICA QUE
$|\mathrm{Hom}(D, A)| = |\mathrm{Hom}(B, A)| \cdot |\mathrm{Hom}(C, A)|$ (★★)
PARA TODO A.

DIGAMOS QUE $B = 21$ E $C = 02$.
ENCONTRE UM D TAL QUE (★★) VALHA PARA TODO A.







$D = 22$?

LA /NOV/2017

HOJE! DUALIDADE - MAS COMEÇANDO UM POUCO ADIANTE DE ONDE A GENTE CHEGOU NA AULA PASSADA...

① $B \xleftarrow{p_1} D \xrightarrow{p_2} C$ É UM

OBS: (B, C, D, p_1, p_2) "PRODUCT DIAGRAM" QUANDO

$\forall A.\ \forall f: A \to B.\ \forall g: A \to C.$
$\exists! h: A \to C.\ (p_1 \circ h = f \wedge p_2 \circ h = g)$

② $B \xrightarrow{p_1} D \xleftarrow{p_2} C$

É UM "COPRODUCT DIAGRAM" QUANDO

$\forall A.\ \forall f: B \to A.\ \forall g: C \to A.$
$\exists! h: C \to A.\ (h \circ p_1 = f \wedge h \circ p_2 = g)$

Em Set,

Se $A = \{0, 1, 2\}$

e $B = \{3, 4, 5, 6\}$,

ENTÃO

$|Hom(A,B)| = 64 = 4^3$

$|Hom(B,A)| = 81 = 3^4$

$|Hom(B,B)| = 256 = 4^4$

$A \xrightarrow{f} B$

$0 \mapsto f(0)$
$1 \mapsto f(1)$
$2 \mapsto f(2)$

$|Hom(X, Y)| = |Y|^{|X|}$

(A) $|Hom(A,B)| \cdot |Hom(A,C)| = |Hom(A,D)|$

Se $|A| = 2$, $|B| = 3$, $|C| = 4$,

$9 \cdot 16 = |D|^{|A|} = |D|^2 \quad |D| = 12$

$|B|^{|A|} \cdot |C|^{|A|}$

$3^2 \cdot 4^2$

USAN

$\lambda A:$

1