

LA 22/AGOSTO/2017

$$\begin{array}{l} a) 2 \cdot (3+4) + 5 \cdot 6 \\ \quad \underbrace{\quad 7 \quad} \quad \underbrace{\quad 30 \quad} \\ \quad \underbrace{\quad 14 \quad} \\ \quad 44 \end{array}$$

$$\begin{array}{l} 2 \cdot (3+4) + 5 \cdot 6 \rightarrow 2 \cdot (3+4) + 30 \\ \downarrow \quad \quad \quad \downarrow \\ 2 \cdot 7 + 5 \cdot 6 \rightarrow 2 \cdot 7 + 30 \\ \downarrow \quad \quad \quad \downarrow \\ 14 + 5 \cdot 6 \rightarrow 14 + 30 \\ \downarrow \\ 44 \end{array}$$

$$\begin{array}{l} b) 2+3+4 \\ \quad \underbrace{\quad 5 \quad} \\ \quad 9 \end{array} \quad \begin{array}{l} 2+3+4 \rightarrow 2+7 \\ \downarrow \quad \quad \downarrow \\ 5+4 \rightarrow 9 \\ 2+3+4 = 2+7 \\ = 9 \end{array}$$

$$\begin{array}{l} a) 2 \cdot (3+4) + 5 \cdot 6 \\ \quad \underbrace{\quad 7 \quad} \quad \underbrace{\quad 30 \quad} \\ \quad \underbrace{\quad 14 \quad} \\ \quad 44 \end{array}$$

$$\begin{array}{l} 2 \cdot (3+4) + 5 \cdot 6 \rightarrow 2 \cdot (3+4) + 30 \\ \downarrow \quad \quad \quad \downarrow \\ 2 \cdot 7 + 5 \cdot 6 \rightarrow 2 \cdot 7 + 30 \\ \downarrow \quad \quad \quad \downarrow \\ 14 + 5 \cdot 6 \rightarrow 14 + 30 \\ \downarrow \\ 44 \end{array}$$

$$\begin{array}{l} b) 2+3+4 \\ \quad \underbrace{\quad 5 \quad} \\ \quad 9 \end{array}$$

$$\begin{array}{l} 2+3+4 \rightarrow 2+7 \\ \downarrow \quad \quad \downarrow \\ 5+4 \rightarrow 9 \\ 2+3+4 = 2+7 = 9 \end{array}$$

$$c) 2+3+4+5$$

$$\begin{array}{l} 2 \cdot (3+4) + 5 \cdot 6 \\ \quad \underbrace{\quad 7 \quad} \quad \underbrace{\quad 30 \quad} \\ \quad \underbrace{\quad 14 \quad} \\ \quad 44 \end{array}$$

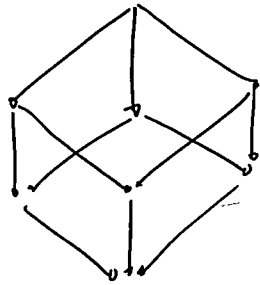
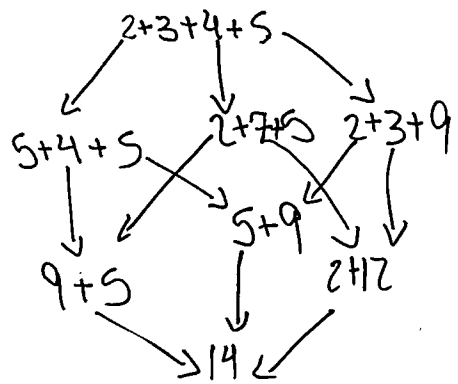
$$\begin{array}{l} 2+3+4 \\ \quad \underbrace{\quad 5 \quad} \\ \quad 9 \end{array}$$

$$\begin{array}{l} 2 \cdot (3+4) + 5 \cdot 6 \rightarrow 2 \cdot (3+4) + 30 \\ \downarrow \quad \quad \quad \downarrow \\ 2 \cdot 7 + 5 \cdot 6 \rightarrow 2 \cdot 7 + 30 \\ \downarrow \quad \quad \quad \downarrow \\ 14 + 5 \cdot 6 \rightarrow 14 + 30 \\ \downarrow \\ 44 \end{array}$$

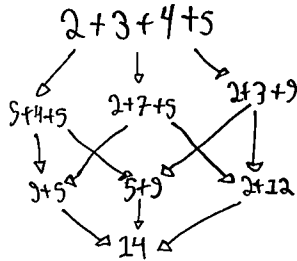
$$\begin{array}{l} 2+3+4 \rightarrow 2+7 \\ \downarrow \quad \quad \downarrow \\ 5+4 \rightarrow 9 \end{array}$$

LA 22/AGOSTO/2017

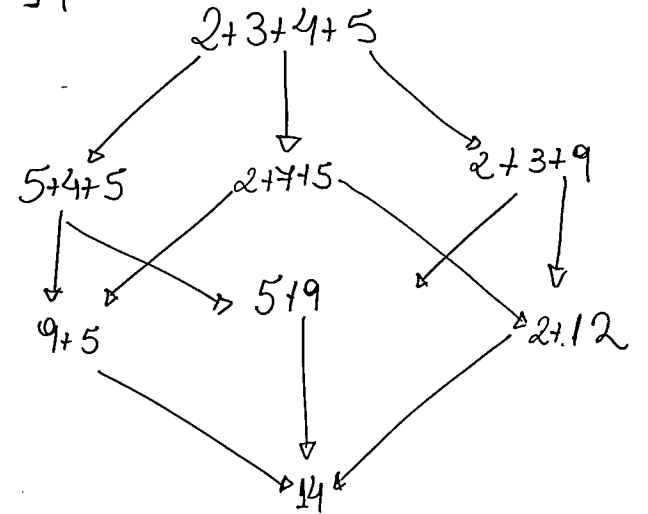
$$c) \underbrace{2+3}_{5} + \underbrace{4+5}_{9} = 14$$



$$c) \underbrace{2+3+4}_{5} + \underbrace{5}_{9} = 14$$



$$2 + \underbrace{3+4}_{7} + 5 = 14$$



LA 22/AGOSTO/2017

O QUE ACONTECERIA SE A GENTE PERMITISSE ALGO COMO

$$\sum_{i \in A} f(i) ?$$

POR EXEMPLO,

$$\sum_{i \in \{2,3,5\}} i = ?$$

$$\downarrow$$

$$2+3+5$$

$$\underbrace{\hspace{2cm}}_5$$

$$\underbrace{\hspace{2cm}}_{10}$$

$$\sum_{i \in \{2,3,3,5\}} i = ?$$

$$\downarrow$$

$$2+3+3+5$$

$$\underbrace{\hspace{1cm}}_5 \quad \underbrace{\hspace{1cm}}_8$$

$$\underbrace{\hspace{2cm}}_{13}$$

$$\{2,3,5\} = \{2,3,3,5\}$$

$$\sum_{i \in \{2,3,5\}} i \neq \sum_{i \in \{2,3,3,5\}} i \quad \parallel$$

$$\sum_{i=2}^5 i \longrightarrow \sum_{i \in \{2,3,4,5\}} i \longrightarrow 2+3+4+5$$

EXERCÍCIOS:

a) $(\lambda a. 10a)(2+3)$

b) $(\lambda a. 10a)((\lambda b. b+4)(3))$

c) $((\lambda a. (\lambda b. 10a+b))(3))(4)$

b) $(\lambda a. 10a)((\lambda b. b+4)(3))$

$$\downarrow$$

$$(\lambda a. 10a)((b+4)[b:=3])$$

$$\downarrow$$

$$(\lambda a. 10a)(3+4)$$

Seja $h = (\lambda a. 10a)$

$$h(3+4) \longrightarrow h(7)$$

$$\downarrow$$

$$(\lambda a. 10a)(3+4) \longrightarrow (\lambda a. 10a)(7)$$

$$\downarrow$$

$$(10a)[a:=3+4] \longrightarrow (10a)[a:=7]$$

$$\downarrow$$

$$10(3+4) \longrightarrow 10 \cdot 7$$

$$\downarrow$$

$$70$$

a) $(\lambda a. 10a)(2+3)$

$$h(2+3) \longrightarrow h(5)$$

$$\downarrow$$

$$(\lambda a. 10a)(2+3) \longrightarrow (\lambda a. 10)(5)$$

$$\downarrow$$

$$(10a)[a:=2+3] \longrightarrow (10a)[a:=5]$$

$$\downarrow$$

$$10(2+3) \longrightarrow 10 \cdot 5$$

$$\downarrow$$

$$50$$

b) $(\lambda a. 10a)((\lambda b. b+4)(3))$

$$\underbrace{\hspace{2cm}}_{(b+4)[b:=3]}$$

$$\underbrace{\hspace{2cm}}_{3+4}$$

$$\underbrace{\hspace{2cm}}_7$$

$$\underbrace{\hspace{2cm}}_{(10a)[a:=7]}$$

$$\underbrace{\hspace{2cm}}_{10 \cdot 7}$$

$$\underbrace{\hspace{2cm}}_{70}$$

b) $(\lambda a. 10a)((\lambda b. b+4)(3))$

$$\underbrace{\hspace{2cm}}_{(b+4)(3)}$$

$$\underbrace{\hspace{2cm}}_{3+4}$$

$$\underbrace{\hspace{2cm}}_7$$

$$\underbrace{\hspace{2cm}}_{10 \cdot 7}$$

$$\underbrace{\hspace{2cm}}_{70}$$

LA 22/AGOSTO/2017

¿QUE ACONTECERIA
SE A GENTE PERMITISSE
ALGO COMO

$$\sum_{i \in A} f(i) ?$$

POR EJEMPLO,

$$\sum_{i \in \{2,3,5\}} i = ?$$

$$\begin{array}{c} \downarrow \\ 2+3+5 \\ \hline 5 \\ \hline 10 \end{array}$$

$$\{2,3,5\} = \{2,3,3,5\}$$

MÁS

$$\sum_{i \in \{2,3,5\}} i \neq \sum_{i \in \{2,3,3,5\}} i \quad ||$$

$$\sum_{i \in \{2,3,3,5\}} i = ?$$

$$\begin{array}{c} \downarrow \\ 2+3+3+5 \\ \hline 5 \quad 8 \\ \hline 13 \end{array}$$

$$\sum_{i=2}^5 i \longrightarrow \sum_{i \in \{2,3,4,5\}} i \longrightarrow 2+3+4+5$$

EXERCÍCIOS:

a) $(\lambda a. 10a)(2+3)$

b) $(\lambda a. 10a)((\lambda b. b+4)(3))$

c) $((\lambda a. (\lambda b. 10a+b))(3))(4)$

d) $((\lambda f. (\lambda a. f(f(a)))(\lambda x. 10x))(7)$

c) $((\lambda a. (\lambda b. 10a+b))(3))(4)$

$$(\lambda b. 10 \cdot 3 + b)(4)$$

$$10 \cdot 3 + 4$$

$$30 + 4$$

$$34$$

d) $((\lambda f. (\lambda a. f(f(a)))(\lambda x. 10x))(7)$

$$((\lambda f. (\lambda a. f(f(a))))(\lambda x. 10x))(7)$$

$$((\lambda a. f(f(a)))(\lambda x. 10x)[f := \lambda x. 10x])$$

$$((\lambda a. (\lambda x. 10x)(\lambda x. 10x)(a)) [a := 7])$$

$$((\lambda x. 10x)(\lambda x. 10x)(7))$$

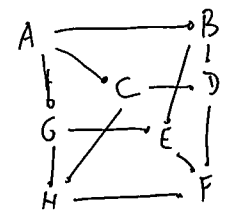
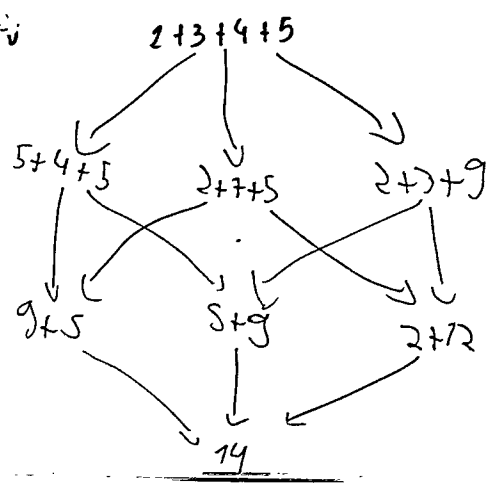
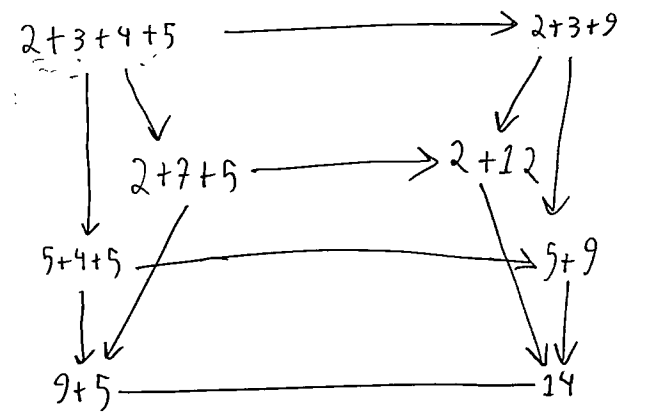
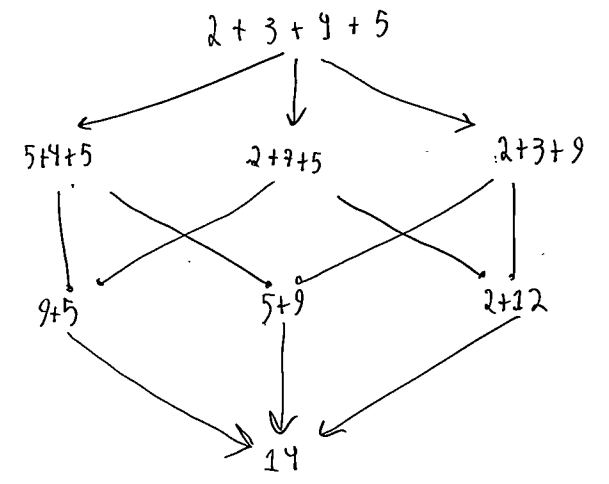
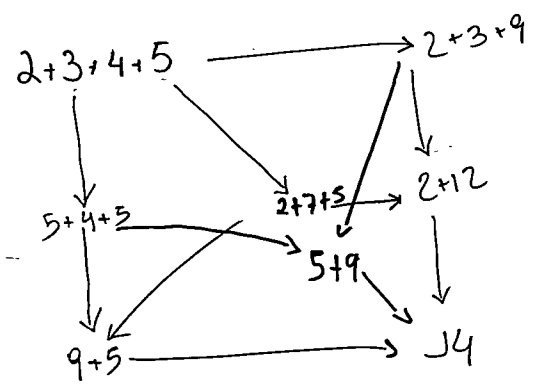
$$((\lambda x. 10x)(10 \cdot 7))$$

$$((\lambda x. 10x)(70))$$

$$(10 \cdot 70)$$

$$700$$

LA 29/A60/2017



LA 29/AGO/2017

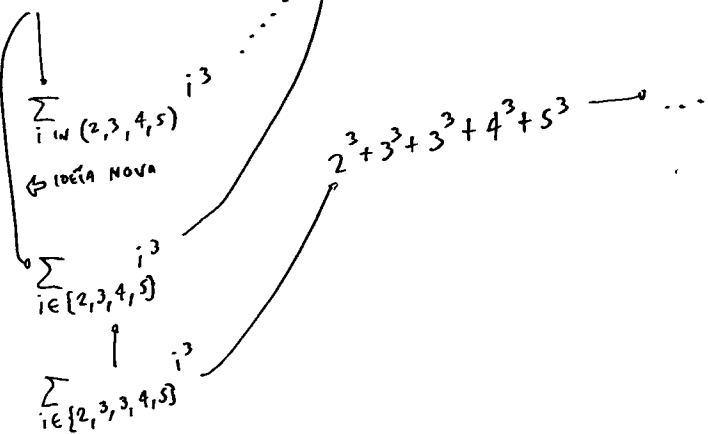
PRINCÍPIO:

TODAS AS SEQUÊNCIAS DE REDUÇÃO "CONVERGEM".

$$(2+3) \cdot (4+5) \rightarrow$$

ESSE PRINCÍPIO VAI VALER SEMPRE.

$$\sum_{i=2}^5 i^3 \rightarrow 2^3 + 3^3 + 4^3 + 5^3 \rightarrow \dots$$



$$(a+b)(a-b) \rightarrow a^2 - b^2$$

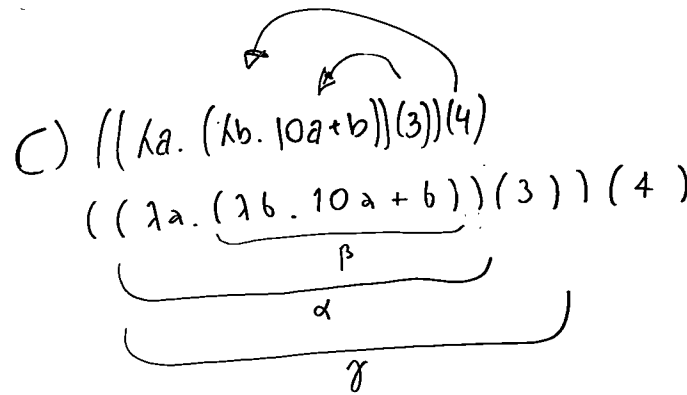
EXERCÍCIOS:

a) $(\lambda a \cdot 10a)(2+3)$

b) $(\lambda a \cdot 10a)((\lambda b \cdot b + 4)(3))$

c) $((\lambda a \cdot (\lambda b \cdot 10a + b))(3))(4)$

d) $((\lambda f \cdot (\lambda a \cdot f(f(a))))(\lambda x \cdot 10x))(7)$



$$\gamma(4) \rightarrow (\alpha(3))(4)$$

$$\begin{aligned} \gamma(4) &= (\alpha(3))(4) \\ &= ((\lambda a \cdot \beta)(3))(4) \\ &= (\beta[a:=3])(4) \\ &= ((\lambda b \cdot 10a + b)[a:=3])(4) \\ &= ((\lambda b \cdot 10 \cdot 3 + b))(4) \\ &= (\lambda b \cdot 30 + b)(4) \\ &\stackrel{OK}{=} (30 + b)[b:=4] \\ &\stackrel{OK}{=} (30 + 4) \\ &\stackrel{OK}{=} (34) \end{aligned}$$

- A = {1, 2}
- B = {3, 4}
- C = {30, 40}
- D = {10, 20}
- f = {(3, 30), (4, 40)}
- g = {(1, 10), (2, 20)}

a) $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$

b) $A \times D = \{(1, 10), (1, 20), (2, 10), (2, 20)\}$

$A \rightarrow D = \left\{ \left\{ \begin{matrix} (1, 10) \\ (1, 20) \end{matrix} \right\}, \left\{ \begin{matrix} (2, 10) \\ (2, 20) \end{matrix} \right\} \right\}$

$$\frac{\frac{p}{\pi p} \frac{\pi' p'}{f(\pi' p)} \text{ app}}{(\pi p, f(\pi' p)) \text{ pair}} \quad \frac{\frac{(1,3) \pi'}{2} \frac{3 \pi'}{30} \text{ app}}{(1,30) \text{ pair}} \quad \left. \begin{matrix} \frac{p}{\pi p} \frac{\pi' p'}{f(\pi' p)} \text{ app} \\ \frac{g(\pi p)}{f(\pi' p)} \text{ pair} \end{matrix} \right\} \frac{p}{\pi p} \frac{\pi' p'}{f(\pi' p)}$$

$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$
 $A \rightarrow D = \left\{ \left\{ \begin{matrix} (1, 10) \\ (2, 10) \end{matrix} \right\}, \left\{ \begin{matrix} (1, 20) \\ (2, 20) \end{matrix} \right\}, \left\{ \begin{matrix} (1, 30) \\ (2, 30) \end{matrix} \right\}, \left\{ \begin{matrix} (1, 40) \\ (2, 40) \end{matrix} \right\} \right\}$

$(\pi p, f(\pi' p)) =$

$\frac{(1,3) \pi'}{2} \frac{3 \pi'}{30} \text{ app}}{(1,30) \text{ pair}}$	$\frac{(1,4) \pi'}{2} \frac{4 \pi'}{40} \text{ app}}{(1,40) \text{ pair}}$
--	--

c)

$p = (1, 3)$ $\frac{\frac{p}{\pi p} \frac{\pi' p'}{f(\pi' p)} \text{ app}}{(\pi p, f(\pi' p)) \text{ pair}}$ $(1, 30)$	$p = (1, 4)$ $\frac{\frac{1,4}{2} \pi' \frac{4 \pi'}{40} \text{ app}}{(1, 40) \text{ pair}}$	$p = (2, 3)$ $\frac{\frac{(2,3) \pi'}{2} \frac{3 \pi'}{30} \text{ app}}{(2, 30) \text{ pair}}$	$p = (2, 4)$ $\frac{\frac{(2,4) \pi'}{2} \frac{4 \pi'}{40} \text{ app}}{(2, 40) \text{ pair}}$
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$(\pi p, f(\pi' p)) =$

$\frac{\frac{p}{\pi p} \frac{\pi' p'}{f(\pi' p)} \text{ app}}{(\pi p, f(\pi' p)) \text{ pair}}$	$\frac{\frac{(2,3) \pi'}{2} \frac{3 \pi'}{30} \text{ app}}{(2, 30) \text{ pair}}$	$\frac{\frac{(2,4) \pi'}{2} \frac{4 \pi'}{40} \text{ app}}{(2, 40) \text{ pair}}$
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d) $\lambda p: A \times B \cdot (\pi p, f(\pi' p)) = \left\{ \begin{matrix} ((1,3), (1,30)), \\ ((1,4), (1,40)), \\ ((2,3), (2,30)), \\ ((2,4), (2,40)) \end{matrix} \right\}$

$\lambda p: A \times B$

Yes!

LA S/ser/2017

$$D \times C = \{(10,30), (10,40), (20,30), (20,40)\}$$

$$A = \{1, 2\}$$

$$B = \{3, 4\}$$

$$C = \{30, 40\}$$

$$D = \{10, 20\}$$

$$f = \{(3,30), (4,40)\}$$

$$g = \{(1,10), (2,20)\}$$

$$a) A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$$

$$b) A \times D = \{(1,10), (1,20), (2,10), (2,20)\}$$

$$A \rightarrow D = \left\{ \left\{ \begin{matrix} (1,10) \\ (2,10) \end{matrix} \right\}, \left\{ \begin{matrix} (1,20) \\ (2,20) \end{matrix} \right\} \right\}$$

$$A \times D \rightarrow D \times C = \{$$

$$p = (2,3)$$

$$p = (2,3)$$

$$p = (2,4)$$

$$\begin{array}{c} \textcircled{F} \\ \frac{\frac{p}{\pi p} \pi}{g(\pi p)} \frac{\frac{p}{\pi p} \pi}{f(\pi p)} \text{app} \\ \frac{2}{20} \frac{3}{30} \text{app} \\ \frac{(2,3)}{(20,30)} \text{pair} \end{array}$$

$$\frac{(2,3) \pi}{2} \frac{(2,3) \pi}{3} \text{app} \\ \frac{20}{30} \text{app} \\ \frac{(2,3)}{(20,30)} \text{pair}$$

$$\frac{(2,3) \pi}{2} \frac{(2,3) \pi}{3} \text{app} \\ \frac{20}{30} \text{app} \\ \frac{(2,3)}{(20,30)} \text{pair}$$

$$\frac{(2,4) \pi}{2} \frac{(2,4) \pi}{4} \text{app} \\ \frac{20}{40} \text{app} \\ \frac{(2,4)}{(20,40)} \text{pair}$$

$$\lambda p: A \times B$$

$$\textcircled{1} (\lambda p: A \times B. (\pi p, f(\pi p))) - \left\{ \begin{matrix} (1,3), (1,40) \\ (1,4), (1,40) \\ (2,3), (2,30) \\ (2,4), (2,40) \end{matrix} \right\}$$

Yes!

$$\textcircled{H} (\lambda p: A \times B. (g(\pi p), f(\pi p)))$$

$$\left\{ \begin{matrix} ((1,3), (10,30)), \\ ((1,4), (10,40)), \\ ((2,3), (20,30)), \\ ((2,4), (20,40)) \end{matrix} \right\}$$

False

$$\frac{\frac{p}{\pi p} \pi}{f(\pi p)} \frac{\frac{p}{\pi p} \pi}{f(\pi p)} \text{app} \quad \frac{(1,3) \pi}{2} \frac{(1,3) \pi}{3} \text{app} \\ \frac{(1,3)}{(1,30)} \text{pair}$$

$$\frac{\frac{p}{\pi p} \pi}{g(\pi p)} \frac{\frac{p}{\pi p} \pi}{f(\pi p)} \text{app} \\ \frac{(1,3)}{(1,30)} \text{pair}$$

$$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$$

$$A \rightarrow D = \left\{ \left\{ \begin{matrix} (1,10) \\ (2,10) \end{matrix} \right\}, \left\{ \begin{matrix} (1,20) \\ (2,20) \end{matrix} \right\}, \left\{ \begin{matrix} (1,30) \\ (2,30) \end{matrix} \right\}, \left\{ \begin{matrix} (1,40) \\ (2,40) \end{matrix} \right\} \right\}$$

$$(\pi p, f(\pi p)) =$$

$$\lambda p: A \times B. (\pi p, f(\pi p)) =$$

$$\left\{ \begin{matrix} ((1,3), (1,30)), \\ ((1,4), (1,40)), \\ ((2,3), (2,30)), \\ ((2,4), (2,40)) \end{matrix} \right\}$$

$$\frac{\frac{p}{\pi p} \pi}{f(\pi p)} \frac{\frac{p}{\pi p} \pi}{f(\pi p)} \text{app} \\ \frac{(1,3)}{(1,30)} \text{pair}$$

LA S/sec7/2017

$$\textcircled{a} \lambda p: A \times C. (f(\pi p), \pi' p)$$

$\underbrace{\quad}_{:A \times C} \quad \underbrace{\quad}_{:A} \quad \underbrace{\quad}_{:C}$
 $\underbrace{\quad}_{:B}$
 $\underbrace{\quad}_{:B \times C}$

$$A \times C \rightarrow B \times C$$

$$\textcircled{b} \lambda q: B \times C. (h(\pi q), (\pi' q))$$

$\underbrace{\quad}_{:B \times C} \quad \underbrace{\quad}_{:B} \quad \underbrace{\quad}_{:C}$
 $\underbrace{\quad}_{:C \rightarrow D}$
 $\underbrace{\quad}_{:D}$
 $\underbrace{\quad}_{:B \times C \rightarrow D}$

$$f: A \rightarrow B$$

$$g: B \times C \rightarrow D$$

$$h: B \rightarrow (C \rightarrow D)$$

$$k: D \rightarrow E$$

$$\textcircled{c} \lambda b: B. \lambda c: C. g(b, c)$$

$\underbrace{\quad}_{:B} \quad \underbrace{\quad}_{:C} \quad \underbrace{\quad}_{:B \times C}$
 $\underbrace{\quad}_{:B \times C}$
 $\underbrace{\quad}_{:D}$
 $\underbrace{\quad}_{:C \rightarrow D}$

$$\textcircled{d} \lambda p: C \rightarrow D. \lambda c: C. k(\varphi c)$$

$\underbrace{\quad}_{:C \rightarrow D} \quad \underbrace{\quad}_{:C} \quad \underbrace{\quad}_{:C \rightarrow D}$
 $\underbrace{\quad}_{:D}$
 $\underbrace{\quad}_{:E}$
 $\underbrace{\quad}_{:C \times E}$
 $\underbrace{\quad}_{:(C \rightarrow D) \rightarrow C \times E}$

$\varphi \circ$

$Z \in \{2, 3, 4\}$
 $Z \in \{1, 2, 3\}$

$$\textcircled{a} (\lambda c) f := \lambda p: A \times C. (f(\pi p), \pi' p)$$

$\underbrace{\quad}_{:A \times C} \quad \underbrace{\quad}_{:A} \quad \underbrace{\quad}_{:C}$
 $\underbrace{\quad}_{:B}$
 $\underbrace{\quad}_{:B \times C}$
 $\underbrace{\quad}_{:A \times C \rightarrow B \times C}$

$$\frac{p: A \times C}{\pi p: A}$$

$$\textcircled{b} h' := \lambda q: B \times C. (h(\pi q), (\pi' q))$$

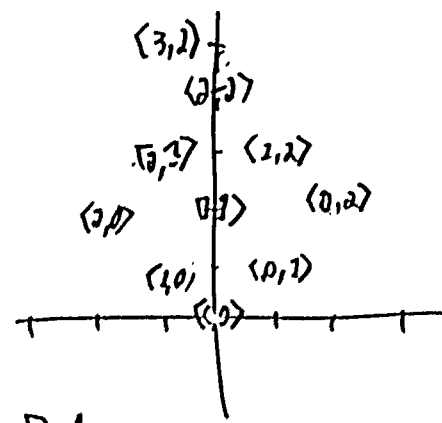
$\underbrace{\quad}_{:B \times C} \quad \underbrace{\quad}_{:B} \quad \underbrace{\quad}_{:C}$
 $\underbrace{\quad}_{:C \rightarrow D}$
 $\underbrace{\quad}_{:D}$
 $\underbrace{\quad}_{:B \times C \rightarrow D}$

$$\textcircled{c} g^\# := \lambda b: B. \lambda c: C. g(b, c)$$

$\underbrace{\quad}_{:B} \quad \underbrace{\quad}_{:C} \quad \underbrace{\quad}_{:B \times C}$
 $\underbrace{\quad}_{:D}$
 $\underbrace{\quad}_{:C \rightarrow D}$
 $\underbrace{\quad}_{:B \rightarrow (C \rightarrow D)}$

LA 12/SET/2017

HOJE: VAMOS INTERRUPTAR
A PROGRAMAÇÃO NORMAL
(λ -CÁLCULO TIPADO) PARA
COMECAR A VER A
RELAÇÃO ENTRE λ -CÁLCULO
E LÓGICA!



a) $\lambda \vdash_n : B.l$

PLANAR HEYTING
ALGEBRAS FOR CHILDREN
→ FIRST PAPER

→ SESSÕES:

1. POSITIONAL NOTATIONS

ESCREVA COMO
CONJUNTO:

2. ZDAGs

Seja $H = \dots$

REPRESENTA $(H, BPM(H))$
COMO GRAFO DIRECIONADO
E COMO PAR DE
CONJUNTOS.

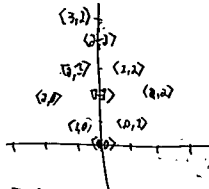
3: LR-COORDINATES

PARA CADA UM DESTES PONTOS
DÊ AS COORDENADAS (x, y) DELE
E REPRESENTA-OS EM \mathbb{Z}_2 .

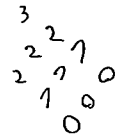
$\langle 0, 0 \rangle, \langle 1, 0 \rangle, \langle 2, 0 \rangle, \langle 0, 1 \rangle, \langle 0, 2 \rangle, \langle 1, 3 \rangle, \langle -1, 0 \rangle, \langle 0, -1 \rangle, \langle -1, -1 \rangle$.

LA 12/SET/2017

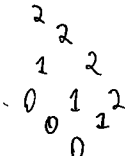
ln	l	n	$l \leq 1$	$n \geq 1$	$ln \leq 11$
00	0	0	1	0	1
10	1	0	1	0	1
01	0	1	1	1	1
20	2	0	0	0	0
11	1	1	1	1	1
02	0	2	1	1	0
21	2	1	0	1	0
12	1	2	1	1	0
22	2	2	0	1	0
32	3	2	0	1	0



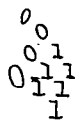
a) $\lambda ln: B.l$



b)



c)



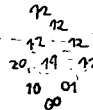
d)



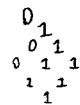
e)



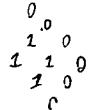
f)



g)



h)



EXERCÍCIO (GRANDE):

Seja $B = \begin{matrix} 32 & 22 & 12 & 02 \\ 20 & 11 & 01 & 00 \end{matrix}$

Calcule, e represente em notação posicional quando possível:

- a) $\lambda lr: B.l$
- b) $\lambda lr: B.r$
- c) $\lambda lr: B. (l \leq 1)$
- d) $\lambda lr: B. (r \geq 1)$
- e) $\lambda lr: B. l \leq 11$
- f) $\lambda lr: B. l \& 12$
- g) $\lambda lr: B. \text{valid}(\langle l+1, r \rangle)$
- h) $\lambda lr: B. lr \text{ leftof } 11$
- i) $\lambda lr: B. lr \text{ leftof } 12$
- j) $\lambda lr: B. lr \text{ above } 11$
- k) $\lambda lr: B. ne(lr)$
- l) $\lambda lr: B. nu(lr)$
- m) $20 \rightarrow 11$
- n) $02 \rightarrow 11$
- o) $22 \rightarrow 11$
- p) $00 \rightarrow 11$
- q) $\lambda lr: B. \neg lr$
- r) $\lambda lr: B. \neg \neg lr$
- s) $\lambda lr: B. (lr = \neg lr)$

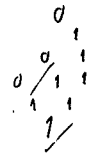
$\frac{00}{12}$

$\frac{00}{00}$

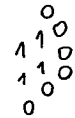
lr	l	r	$lr \text{ leftof } 11$	$\text{valid}(\langle l+1, r \rangle)$
00	0	0	0	1
01	0	1	0	1
02	0	2	0	1
10	1	0	1	1
11	1	1	1	1
12	1	2	0	1
20	2	0	1	0
21	2	1	1	0
22	2	2	0	1
32	3	2	0	0

$\Omega = B$

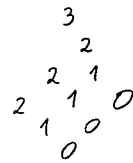
g) $\lambda lr: B. \text{valid}(\langle l+1, r \rangle)$



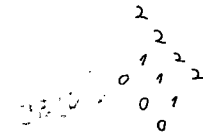
h) $\lambda lr: B. lr \text{ leftof } 11$



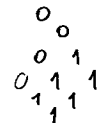
a) $\lambda ln: B.l$



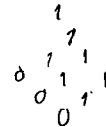
b) $\lambda lr: B.r$



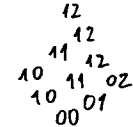
c) $\lambda lr: B. (l \leq 1)$



d) $\lambda lr: B. (r \geq 1)$



f) $\lambda lr: B. l \& 12$



$l \geq l'$
 $\&$
 $n \leq n'$

$LR \& n'$	$\text{valid}(\langle l+1, n \rangle)$	$\text{leftof } 11$
00	1	10
10	1	10
01	1	11
20	0	20
11	1	11
02	1	11
12	0	11
21	1	21
12	1	21
22	1	21
32	0	31