

LA 20/MAR/2017

$$\begin{aligned} a) 2 \cdot (3+4) + 5 \cdot 6 &= 2 \cdot (7) + 5 \cdot 6 \\ &= 14 + 5 \cdot 6 \\ &= 14 + 30 \\ &= 44 \end{aligned}$$

$$2 \cdot (3+4) \stackrel{=}{=} 2 \cdot 3 + 2 \cdot 4$$

$$\begin{aligned} &2 \cdot (3+4) + 5 \cdot 6 \\ &\quad \underbrace{2 \cdot (7)} + 30 \\ &\quad \underbrace{14} + 30 \\ &\quad \underbrace{44} \end{aligned}$$

$$\begin{aligned} &2 \cdot (3+4) + 5 \cdot 6 \\ &\quad \underbrace{2 \cdot 7} + \underbrace{5 \cdot 6} \\ &\quad \underbrace{14} + \underbrace{30} \\ &\quad \underbrace{44} \end{aligned}$$

$$\begin{aligned} 2 \cdot (3+4) + 30 &\leftarrow 2 \cdot (3+4) + 5 \cdot 6 \\ &\quad \downarrow \\ &2 \cdot (7) + 5 \cdot 6 \rightarrow 14 + 5 \cdot 6 \\ &\quad \downarrow \qquad \downarrow \\ &2 \cdot (7) + 30 \rightarrow 14 + 30 \rightarrow 44 \end{aligned}$$

$$\begin{aligned} &2 \cdot (3+4) + 5 \cdot 6 \\ &\quad \underbrace{2 \cdot 7} + \underbrace{5 \cdot 6} \\ &\quad \underbrace{14} + 30 \\ &\quad \underbrace{44} \end{aligned}$$

$$\begin{aligned} &2 \cdot (3+4) + 5 \cdot 6 \\ &\quad \underbrace{2 \cdot 7} + 30 \\ &\quad \underbrace{14} + 30 \\ &\quad \underbrace{44} \end{aligned}$$

LA 20/MAR/2017

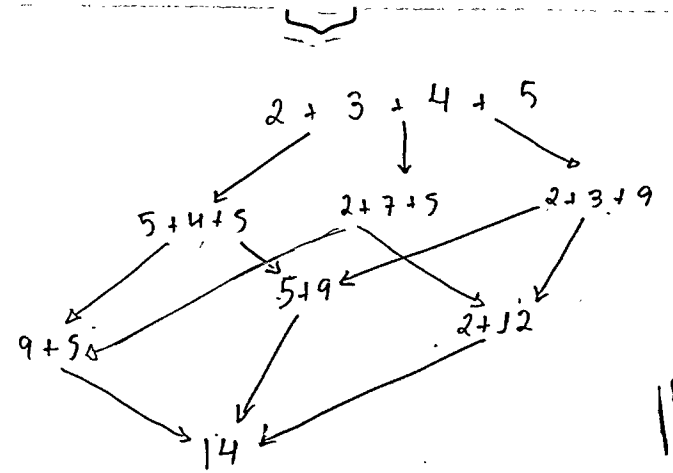
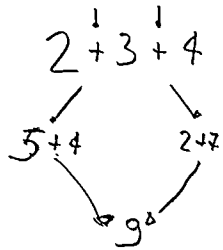
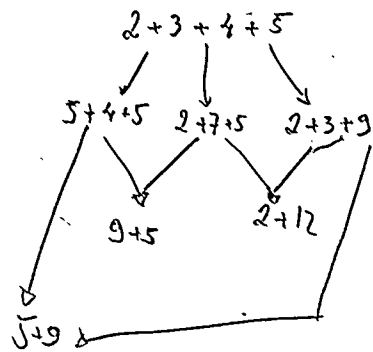
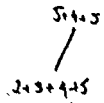
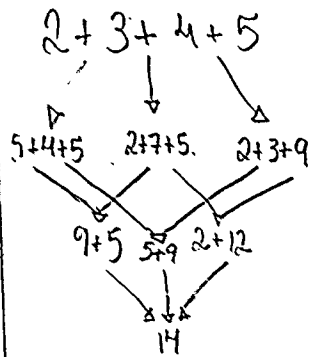
$$\begin{array}{c}
 2 + 3 + 4 \\
 \underbrace{\hspace{1.5cm}}_5 \\
 \underbrace{\hspace{3.5cm}}_9
 \end{array}$$

$$\begin{array}{c}
 3 + 2 + 4 \\
 \underbrace{\hspace{1.5cm}}_5 \\
 \underbrace{\hspace{3.5cm}}_9
 \end{array}$$

$$\begin{array}{c}
 3 + \underbrace{2 + 4}_6 \\
 \underbrace{\hspace{3.5cm}}_9
 \end{array}$$

$$\begin{array}{c}
 2 + 3 + 4 \rightarrow 2 + 7 \\
 \downarrow \qquad \qquad \downarrow \\
 5 + 4 \rightarrow 9
 \end{array}$$

$$\begin{array}{c}
 3 + 2 + 4 \rightarrow 3 + 6 \\
 \downarrow \qquad \qquad \downarrow \\
 5 + 4 \rightarrow 9
 \end{array}$$



LA 20/MAR/2017

a) $(\lambda a. 10a)(2+3)$

b) $(\lambda a. 10a)((\lambda b. b+4)(3))$

$(\lambda a. 10a)(2+3)$

a

$$\begin{aligned} & \underbrace{(\lambda a. 10a)(2+3)} \\ & (10a) [a := (2+3)] \\ & \underbrace{10(2+3)} \\ & \underbrace{10 \cdot 5} \\ & \underbrace{50} \end{aligned}$$

$$\begin{aligned} & (\lambda a. 10a)((\lambda b. b+4)(3)) \\ & \underbrace{((b+4)[b := 3])} \\ & \quad \underbrace{3+4} \\ & \quad \quad \underbrace{7} \\ & \underbrace{10a [a := 7]} \\ & \quad \underbrace{10(7)} \\ & \quad \quad \underbrace{70} \end{aligned}$$

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$$c) ((\lambda a. (\lambda b. 10 \cdot a + b)) (3)) (4)$$

$$(\lambda b. 10 \cdot a + b) [a := 3]$$

$$(\lambda b. 10 \cdot 3 + b)$$

$$(10 \cdot 3 + b) [b := 4]$$

$$10 \cdot 3 + 4$$

$$30$$

$$34$$

$$((\lambda f. (\lambda a. f(f(a)))) (\lambda x. 10x)) (7)$$

$$(\lambda a. f(f(a))) [f := \lambda x. 10x]$$

$$\lambda a. (\lambda x. 10x) (\lambda x. 10x(a))$$

$$((\lambda f. (\lambda a. f(f(a)))) (\lambda x. 10x)) (7)$$

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$$c) ((\lambda a. (\lambda b. 10a + b)) (3)) (4)$$

$$(\lambda b. 10 \cdot a + b) [a := 3]$$

$$(\lambda b. 10 \cdot 3 + b)$$

$$(10 \cdot 3 + b) [b := 4]$$

$$10 \cdot 3 + 4$$

$$30$$

$$34$$

$$((\lambda f. (\lambda a. f(f(a)))) (\lambda x. 10x)) (7)$$

$$c) ((\lambda a. (\lambda b. 10 \cdot a + b)) (3)) (4)$$

$$\lambda b. 10 \cdot 3 + b$$

$$10 \cdot 3 + 4$$

$$34$$

((

$$((\lambda f. (\lambda a. f(f(a)))) (\lambda x. 10x)) (7)$$

$$c) ((\lambda a. (\lambda b. 10 \cdot a + b)) (3)) (4)$$

$$\lambda b. 10 \cdot 3 + b$$

$$10 \cdot 3 + 4$$

$$34$$

LA 27/MAR/2018

$$\begin{aligned} c) & ((\lambda a. (\lambda b. 10a + b)) (3)) (4) \\ & \underbrace{(\lambda b. 10a + b)[a:=3]} \\ & \underbrace{(\lambda b. 10 \cdot 3 + b)} \\ & \underbrace{(10 \cdot 3 + b)[b:=4]} \\ & \underbrace{10 \cdot 3 + 4} \\ & \underbrace{30} \\ & \underbrace{34} \end{aligned}$$

$$d. ((\lambda f. (\lambda a. F(F(a)))) (\lambda x. 10x)) (7)$$

$$\begin{aligned} & \underbrace{(\lambda a. F(F(a)))[f := (\lambda x. 10 \cdot x)]} \\ & \underbrace{(\lambda a. (\lambda x. 10 \cdot x)((\lambda x. 10 \cdot x)(a)))} \\ & \underbrace{(\lambda a. (10 \cdot 10 \cdot a))} \\ & \underbrace{10 \cdot 10 \cdot 7} \\ & \underbrace{700} \\ & \underbrace{(\lambda x. 10x)(7)} \\ & \underbrace{(\lambda x. 10 \cdot x)((\lambda x. 10x)(7))} \\ & \underbrace{70} \\ & \underbrace{70} \\ & \underbrace{700} \end{aligned}$$

LA 3/ABRIL/2018

$$\textcircled{a} \underbrace{(\lambda a \cdot 10a)}_{\alpha} \underbrace{(2+3)}_{\beta}$$

γ

$$\begin{aligned} \alpha(\beta) &= \alpha\beta \\ &= a(2+3) \\ &= a(5) \\ &= (\lambda a \cdot 10a)(5) \\ &= (10a)[a:=5] \\ &= (10 \cdot 5) \\ &= 50 \end{aligned}$$

$$\textcircled{c} \underbrace{((\lambda a. (\lambda b. 10a+b)) (3)) (4)}_{\gamma}$$

$$\begin{aligned} \gamma &= \beta 4 \\ &= (\alpha 3) 4 \\ &= ((\lambda a. \lambda b. 10a+b) 3) 4 \\ &= (\lambda b. 10 \cdot 3 + b) 4 \\ &= (\lambda b. 30 + b) [b:=4] \\ &= (30 + 4) \\ &= 34 \end{aligned}$$

$$((\lambda b. (\lambda a. f(f(a)))) (\lambda x. 10x)) (7)$$

$$\underbrace{\underbrace{((\lambda f. (\lambda a. f(f(a))))}_{\alpha}}_{\beta}}_{\gamma} (\lambda x. 10x)) (7)$$

$$\begin{aligned} \gamma(z) &= \gamma(\alpha(\beta)) \\ &= \gamma((\lambda f. (\lambda a. f(f(a)))) (\lambda x. 10x)) \end{aligned}$$

LA 3/ABRIL/2018

P. 10:

$$S \Rightarrow A_m: A = \{1, 2\},$$

$$B = \{3, 4\},$$

$$C = \{30, 40\},$$

$$D = \{10, 20\},$$

$$f = \left\{ \begin{array}{l} (3, 30) \\ (4, 40) \end{array} \right\},$$

$$g = \left\{ \begin{array}{l} (1, 10) \\ (2, 20) \end{array} \right\}.$$

1) $A \times B = ?$

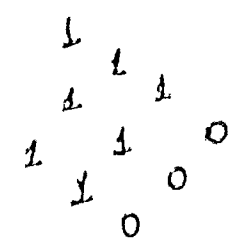
2) $A \rightarrow D = ?$

$$A \times B = \left\{ \begin{array}{l} (1,3), (1,4) \\ (2,3), (2,4) \end{array} \right\}$$

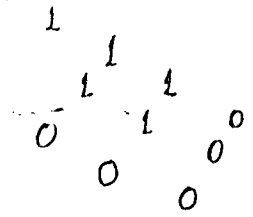
$$A \rightarrow D = \left\{ \begin{array}{l} \{(1,10)\}, \{(2,20)\}, \{(1,10)\}, \{(2,20)\} \\ \{(1,10)\}, \{(2,20)\}, \{(1,10)\}, \{(2,20)\} \\ \{(1,10)\}, \{(2,20)\}, \{(1,10)\}, \{(2,20)\} \\ \{(1,10)\}, \{(2,20)\}, \{(1,10)\}, \{(2,20)\} \end{array} \right\}$$

LA 27/11/10/2018

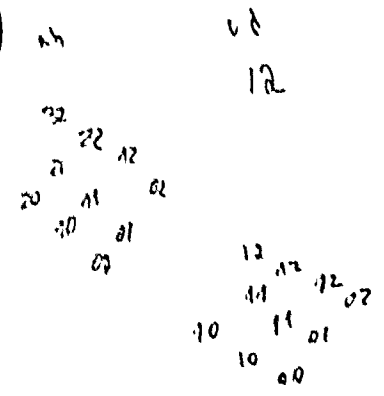
① λ lr: B.l.r left of 12



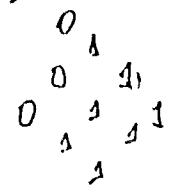
② λ lr: B.l.r same as 12



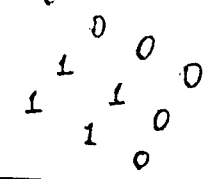
F)



③ λ lr: B.l.r with $(l+1, r)$



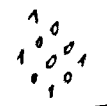
④ λ lr: B.l.r with a, b, c, d



l) $mw(00) = mw(10) = mw(20) = 20$
 $mw(01) = mw(11) = mw(21) = 21$
 $mw(02) = mw(12) = mw(22) = mw(32) = 32$

k) $me(00) = me(01) = me(02) = 02$
 $me(10) = me(11) = me(12) = 12$
 $me(20) = me(21) = me(22) = 22$
 $me(32) = 32$

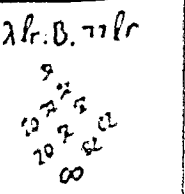
r) $(00) \rightarrow (00) : \sup(B) = 32$
 $(02) \rightarrow (00) : mw(00) = 20$
 $(20) \rightarrow (00) : me(00) = 02$
 $(32) \rightarrow (00) : 00$



m) $20 \rightarrow 11$
 $me(11) = me(12) = 12$
n) $02 \rightarrow 11$
 $mw(11) = mw(21) = 21$
o) $22 \rightarrow 11 = (11)$
p) $00 \rightarrow 11$
 $\sup(B) = 32$

⑤ λ lr: B.l.r

$(00) \rightarrow (00) : \sup(B) = 32$
 $(01) \rightarrow (00) : mw(00) = 20$
 $(02) \rightarrow (00) : mw(00) = 20$
 $(10) \rightarrow (00) : me(00) = 02$
 $(11) \rightarrow (00) : (00)$
 $(12) \rightarrow (00) : (00)$
 $(20) \rightarrow (00) : me(00) = 02$
 $(21) \rightarrow (00) : (00)$
 $(22) \rightarrow (00) : (00)$
 $(32) \rightarrow (00) : (00)$

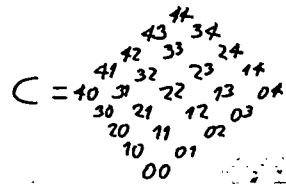
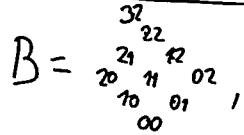


$10 = 7710$
 $\frac{02}{20}$
 $\frac{0}{0}$

$\frac{77}{32}$
 $\frac{00}{00}$

$\frac{32}{22}$
 $\frac{22}{12}$
 $\frac{22}{12}$
 $\frac{02}{02}$
 $\frac{02}{02}$
 \leftarrow

LA 5/JUN/2018



EXERCÍCIOS NOVOS:

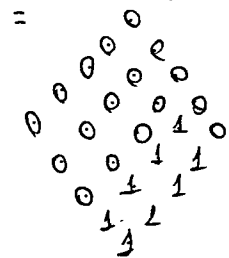
t) $\lambda P : C. (P \rightarrow 22)$

u) $\lambda Q : C. (22 \rightarrow Q)$

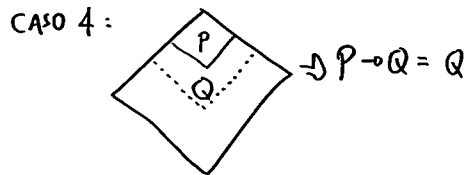
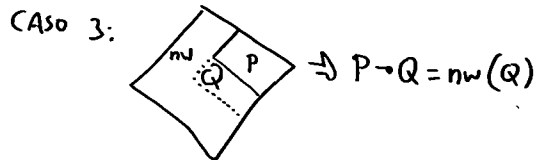
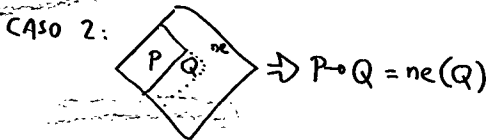
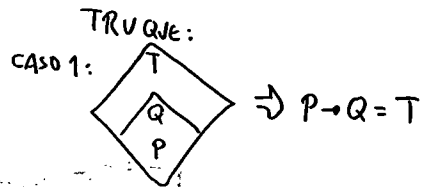
v) ENCONTRE X TAL QUE

$(\lambda P : C. P \leq X) = (\lambda P : C. (P \leq 22) \wedge (P \leq 13))$

$(\lambda P : C. P \leq 13)$



$\frac{a}{1} \frac{b}{2} \leq \frac{c}{1} \frac{d}{3} = \frac{1}{1}$
 $\frac{a}{1} \leq \frac{c}{1} \wedge \frac{b}{2} \leq \frac{d}{3}$



$00 \rightarrow 22 = 44$

$01 \rightarrow 22 = 44$

$02 \rightarrow 22 = 44$

$03 \rightarrow 22 = 42$

$04 \rightarrow 22 = 42$

$10 \rightarrow 22 = 44$

$11 \rightarrow 22 = 44$

$12 \rightarrow 22 = 44$

$13 \rightarrow 22 = 42$

$14 \rightarrow 22 = 42$

$20 \rightarrow 22 = 44$

$21 \rightarrow 22 = 44$

$22 \rightarrow 22 = 44$

$23 \rightarrow 22 = 42$

$24 \rightarrow 22 = 42$

$30 \rightarrow 22 = 24$

$31 \rightarrow 22 = 24$

$32 \rightarrow 22 = 24$

$33 \rightarrow 22 = 22$

$34 \rightarrow 22 = 22$

