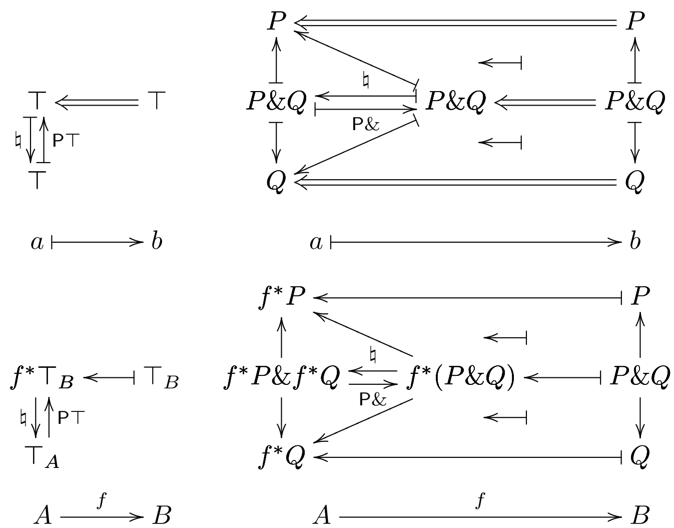
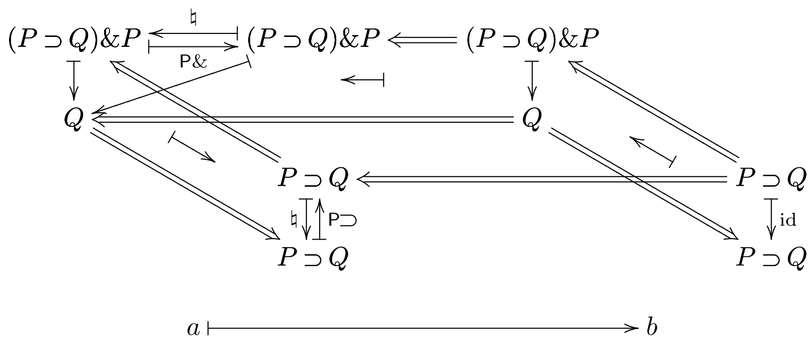
These notes are being changed!!!

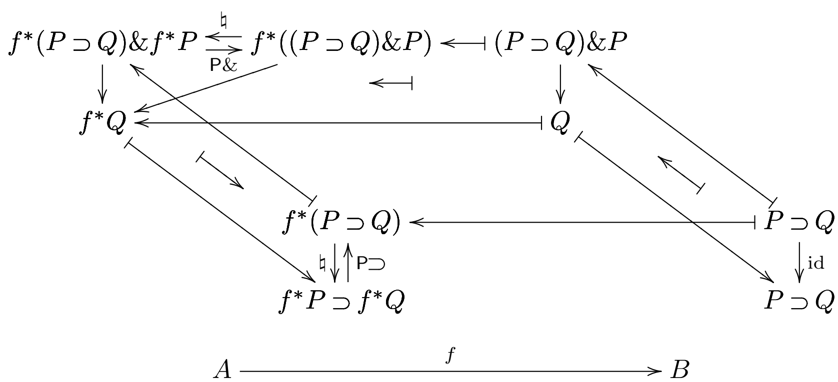
Index of the slides:

Preservation of ‘true’ and ‘and’

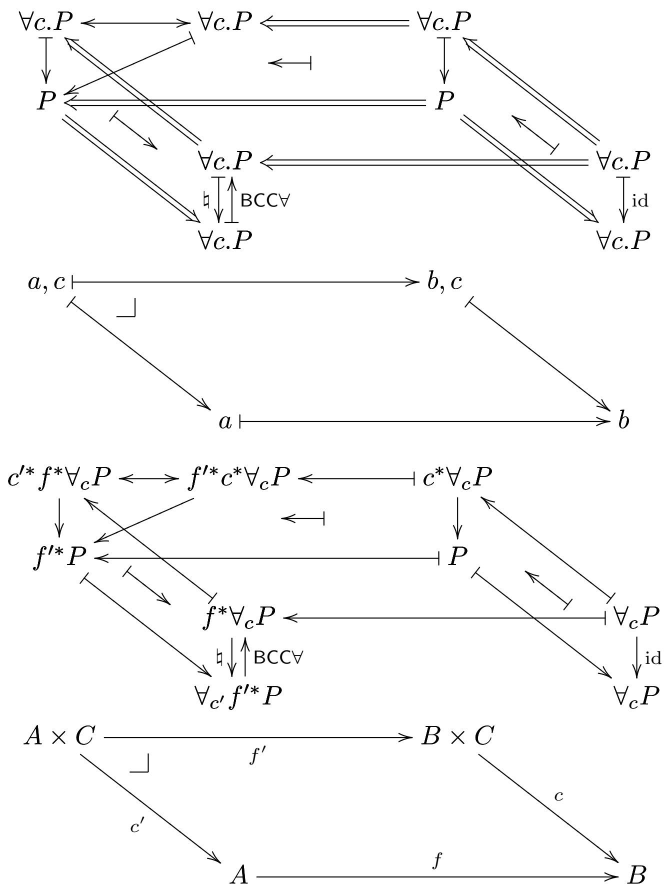


Preservation of 'implies'

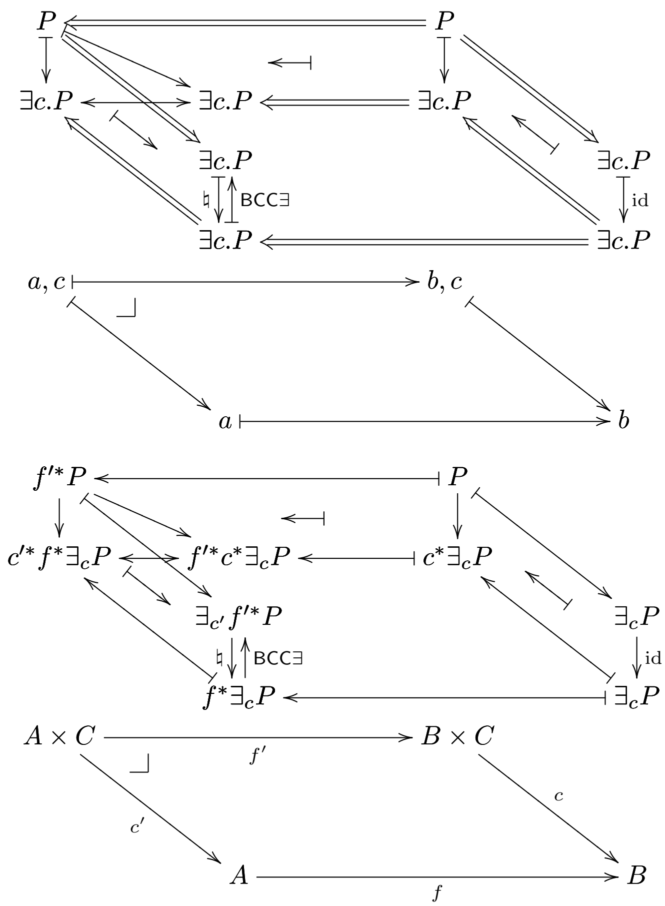




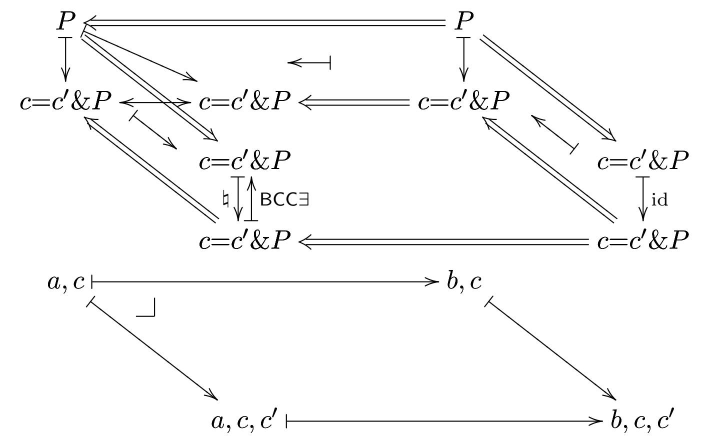
BCC for 'forall'

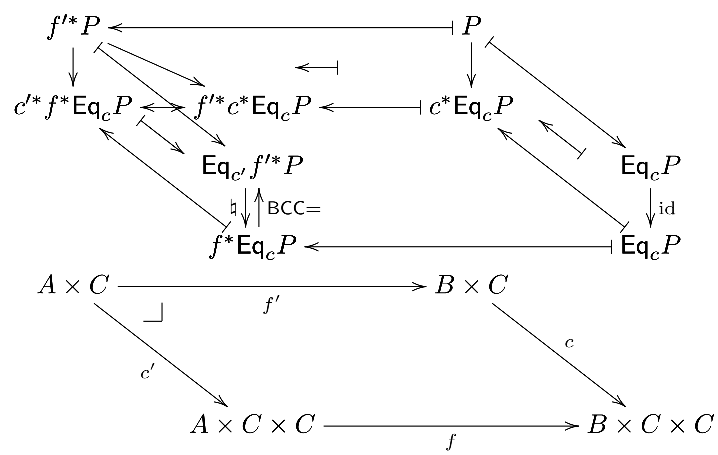


BCC for 'exists'

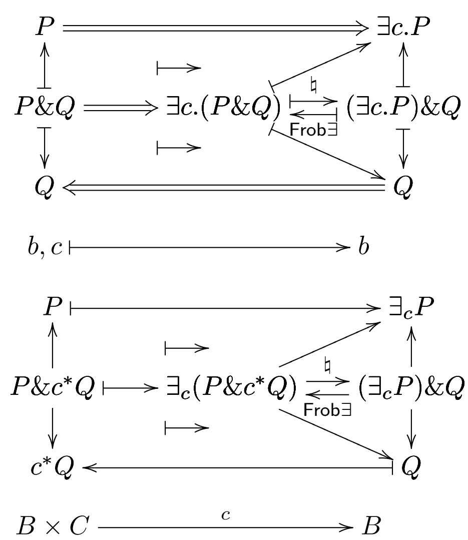


BCC for equality

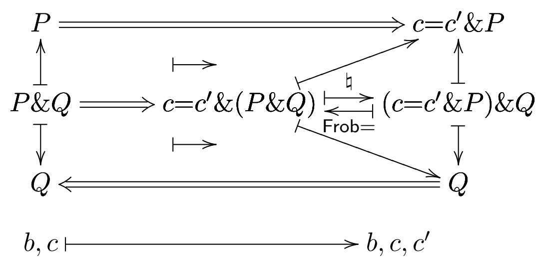


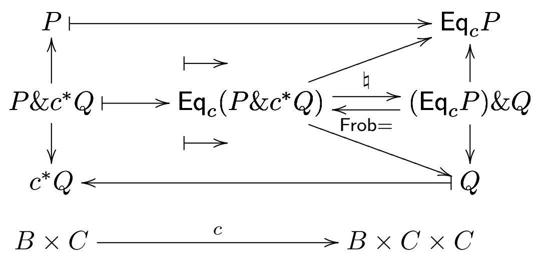


Frobenius for 'exists'



Frobenius for equality



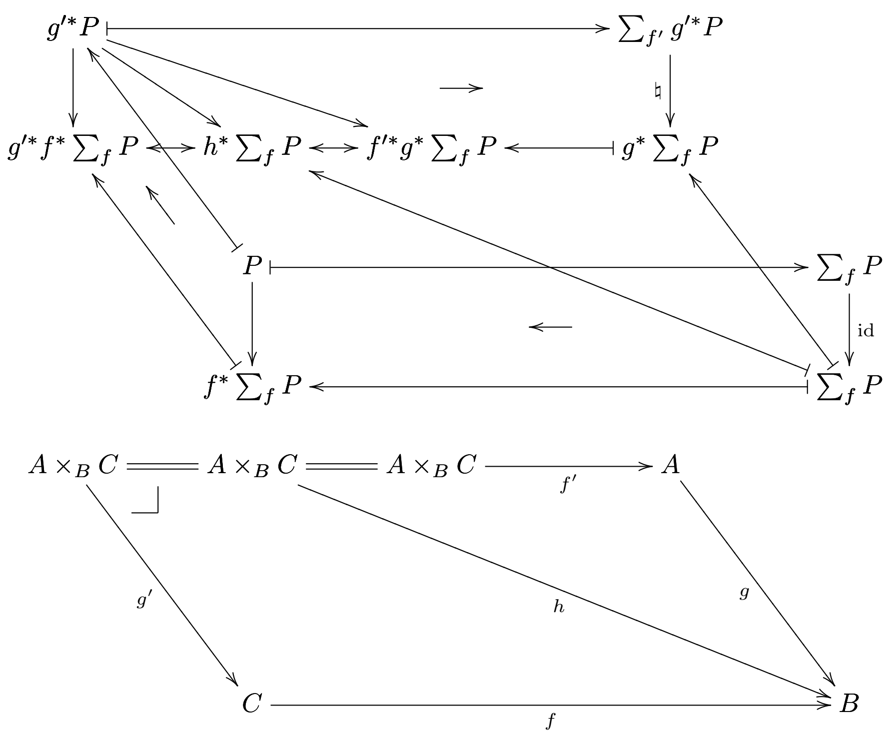


BCC, categorically
This is how the Beck-Chevalley condition (for dependent sums; there is also a variation for dependent products, that we will see soon) is usually stated:

> "If the square formed by f, g, f', g' in the base category in the diagram below is a pullback, and if P is an object over C, then the natural map from $\sum_{f'} g'^* P$ to $g^* \sum_f P$ is an isomorphism."

The upper part of the diagram below shows how to build the map $\natural : \sum_{f'} g'^* P \to g^* \sum_f P$.

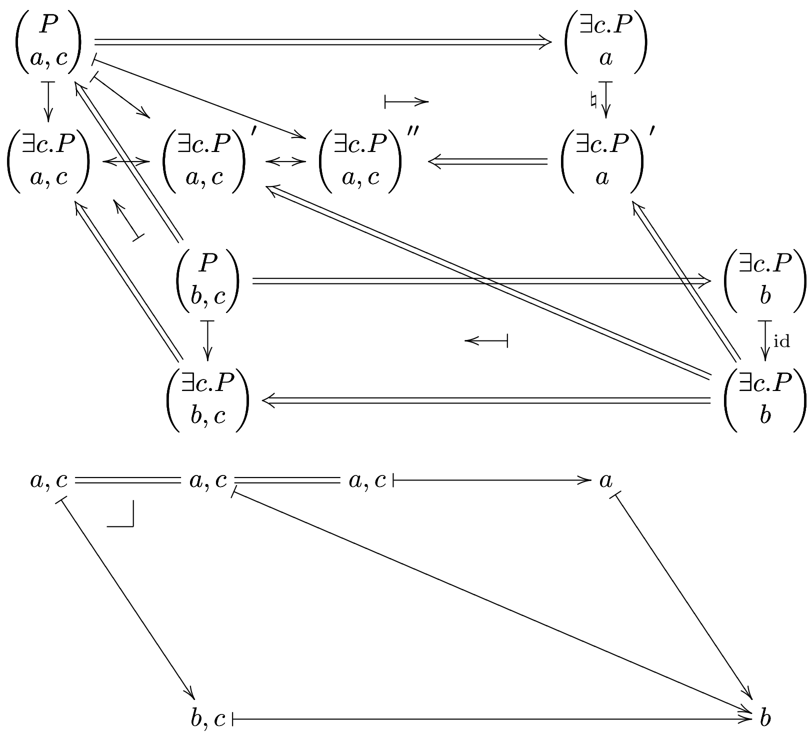
But what does that *mean*?



BCC in a hyperdoctrine

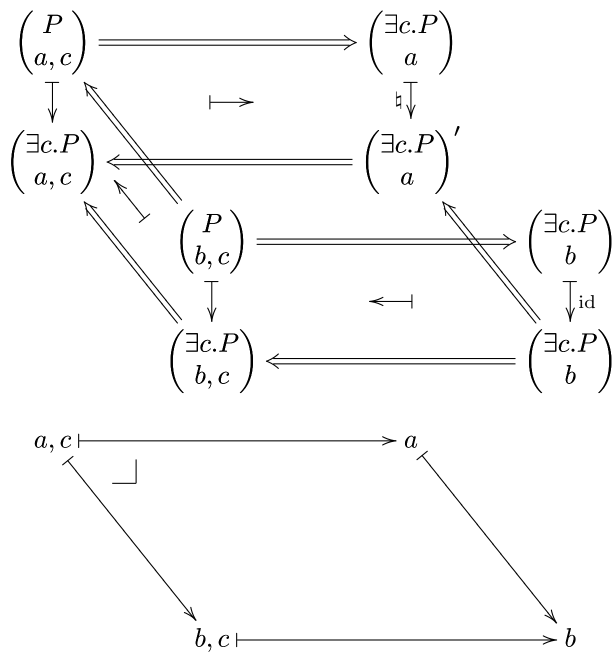
In the case of a hyperdoctrine that means that from an object $\{\, b, c \mid\mid P \,\}$ and a map $a \mapsto b$ there are two ways to build an object that deserves the name "$\{\, a \mid\mid \exists c.P \,\}$"...

...and without BCC we would know a map between them going in one direction, $\natural : \{\, a \mid\mid \exists c.P \,\} \to \{\, a \mid\mid \exists c.P \,\}'$, but we wouldn't know that it is an iso.

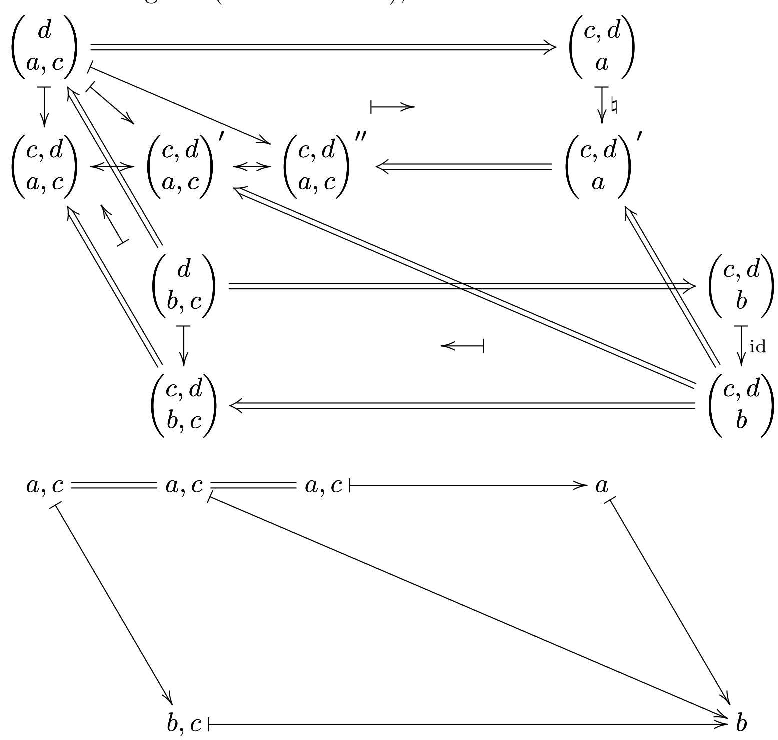


BCC: collapsing isomorphic objects

One trick to make the previous (big) diagram simpler to draw is to draw the objects that are isomorphic and "deserve the same name" — but that may be different — as a single object; these collapsed objects (here just $\{\, a, c \mid\mid \exists c.P \,\}$) have more than one functor arrow pointing to them, which indicates that they have more than one construction.

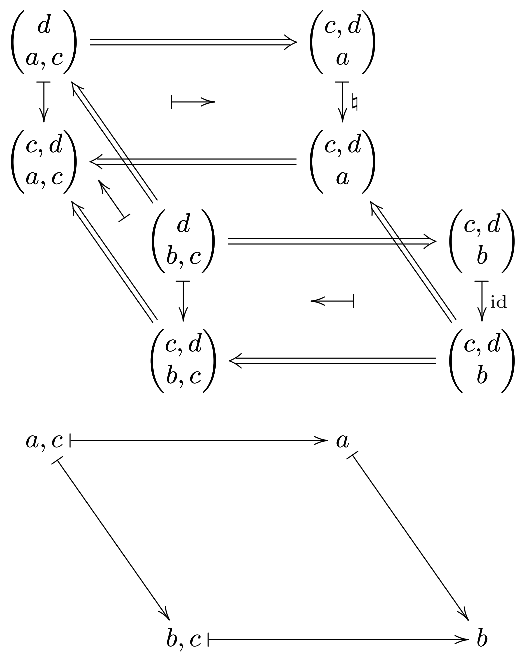


BCC for dependent sums: full diagram
BCC: full diagram (no isos hidden), in $\mathbf{Set}^{\rightarrow}$.



BCC: smaller diagram

BCC: smaller diagram.



BCC: smaller diagrams

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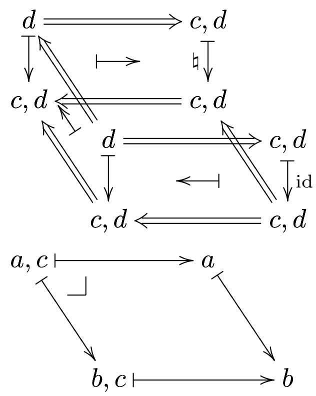
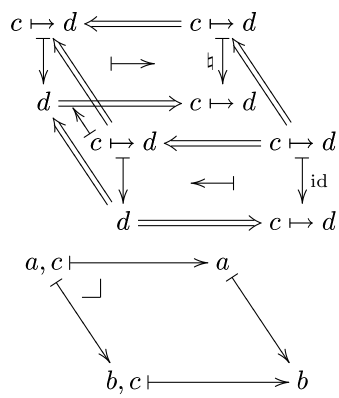
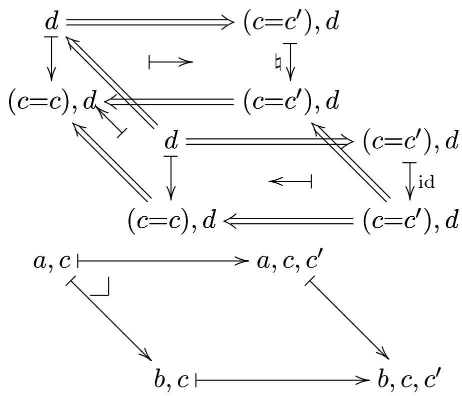


$$\dfrac{\dfrac{a \vdash b \quad \dfrac{\dfrac{\dfrac{b, c \vdash D}{b \vdash \Sigma c.D}}{b; p \vdash p}}{b, c; d \vdash \langle c, d\rangle}}{a, c; d \vdash \langle c, d\rangle}}{a; p \vdash \langle c, d\rangle[c, d := p]}$$



$$\dfrac{\dfrac{a \vdash b \quad \dfrac{\dfrac{\dfrac{b, c \vdash D}{b \vdash \Pi c.D}}{b; f \vdash f}}{b, c; f \vdash fc}}{a, c; f \vdash fc}}{a; f \vdash \lambda c.fc}$$

BCC: trees



$$\dfrac{\dfrac{a, c \vdash b, c \qquad \dfrac{\dfrac{\dfrac{b, c \vdash D}{b, c, c' \vdash \Sigma(c{=}c').D}}{b, c, c'; ((c{=}c'), d) \vdash ((c{=}c').d)}}{b, c; d \vdash \langle (c{=}c), d\rangle}}{a, c; d \vdash \langle (c{=}c), d\rangle}}{a, c; ((c{=}c'), d) \vdash \langle (c{=}c'), d\rangle}$$