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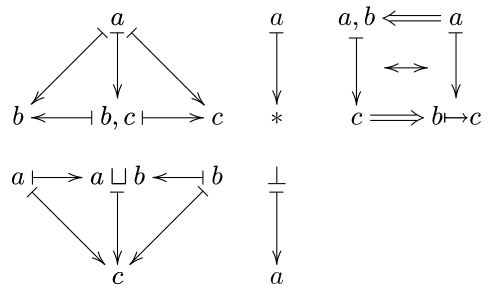
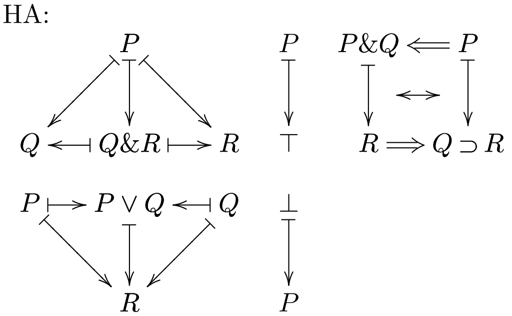
The BiCCC structure and the classifier axiom

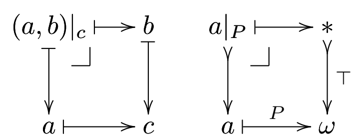
CCC:



HA:



Topos:



The pre-hyperdoctrine structure

Hyperdoctrine:

$$\begin{pmatrix}P\\a\end{pmatrix} \underset{\Longleftarrow}{\overset{\square}{\longmapsto}} \begin{pmatrix}P\\b\end{pmatrix} \qquad \begin{array}{ccc} \begin{pmatrix}P\\a,b\end{pmatrix} & \overset{\mathrm{co}\square}{\Longmapsto} & \begin{pmatrix}b{=}b'\&P\\a,b,b'\end{pmatrix} \\ \downarrow & \leftrightarrow & \downarrow \\ \begin{pmatrix}Q[b,b]\\a,b\end{pmatrix} & \underset{\Longleftarrow}{\overset{\square}{\longmapsto}} & \begin{pmatrix}Q[b,b']\\a,b,b'\end{pmatrix} \end{array} \qquad \begin{array}{ccc} \begin{pmatrix}P\\a,b\end{pmatrix} & \overset{\mathrm{co}\square}{\Longmapsto} & \begin{pmatrix}\exists b.P\\a\end{pmatrix} \\ \downarrow & \leftrightarrow & \downarrow \\ \begin{pmatrix}Q\\a,b\end{pmatrix} & \underset{\Longleftarrow}{\overset{\square}{\longmapsto}} & \begin{pmatrix}Q\\a\end{pmatrix} \\ \downarrow & \leftrightarrow & \downarrow \\ \begin{pmatrix}R\\a,b\end{pmatrix} & \Longrightarrow & \begin{pmatrix}\forall b.R\\a\end{pmatrix} \end{array}$$

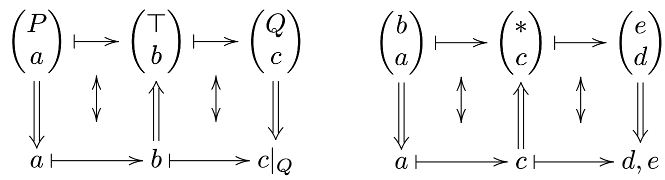
$$a \longmapsto b \qquad a,b \longmapsto a,b,b' \qquad a,b \longmapsto a$$

LCCC:

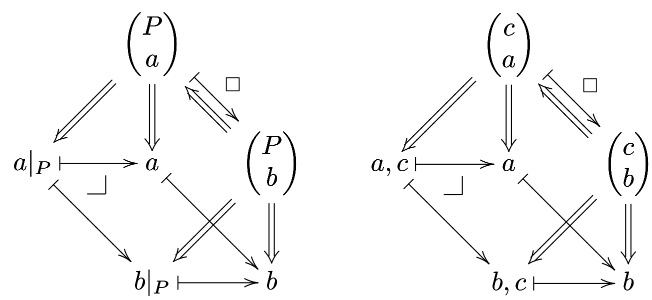
$$\begin{pmatrix}c\\a\end{pmatrix} \underset{\Longleftarrow}{\overset{\square}{\longmapsto}} \begin{pmatrix}c\\b\end{pmatrix} \qquad \begin{array}{ccc} \begin{pmatrix}c\\a,b\end{pmatrix} & \overset{\mathrm{co}\square}{\Longmapsto} & \begin{pmatrix}(b{=}b'),c\\a,b,b'\end{pmatrix} \\ \downarrow & \leftrightarrow & \downarrow \\ \begin{pmatrix}d\\a,b\end{pmatrix} & \underset{\Longleftarrow}{\overset{\square}{\longmapsto}} & \begin{pmatrix}d\\a,b,b'\end{pmatrix} \end{array} \qquad \begin{array}{ccc} \begin{pmatrix}c\\a,b\end{pmatrix} & \overset{\mathrm{co}\square}{\Longmapsto} & \begin{pmatrix}b,c\\a\end{pmatrix} \\ \downarrow & \leftrightarrow & \downarrow \\ \begin{pmatrix}d\\a,b\end{pmatrix} & \underset{\Longleftarrow}{\overset{\square}{\longmapsto}} & \begin{pmatrix}d\\a\end{pmatrix} \\ \downarrow & \leftrightarrow & \downarrow \\ \begin{pmatrix}e\\a,b\end{pmatrix} & \Longrightarrow & \begin{pmatrix}b{\mapsto}e\\a\end{pmatrix} \end{array}$$

$$a \longmapsto b \qquad a,b \longmapsto a,b,b' \qquad a,b \longmapsto a$$

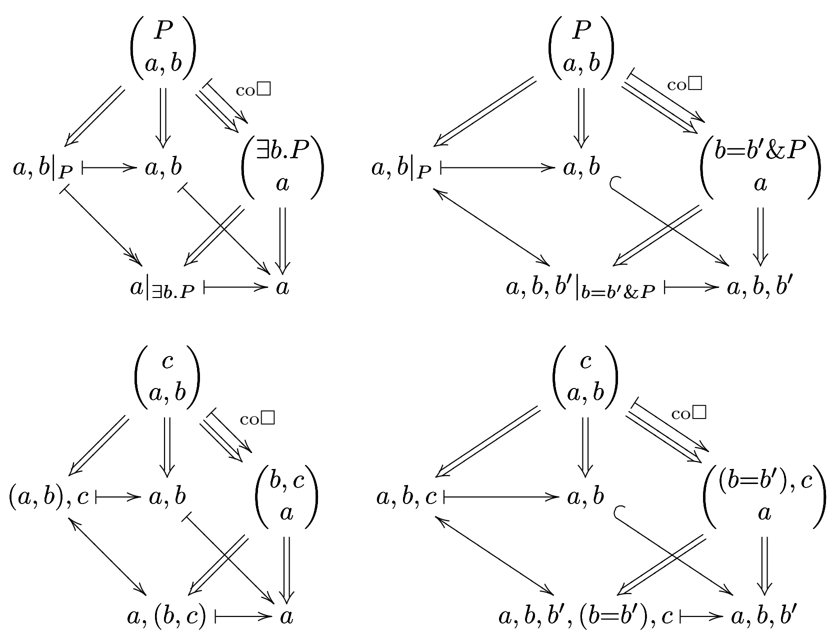
Two CCompC structures in a topos
The two CCompC structures in a topos:



Cartesian morphisms project into pullbacks:



Cocartesian morphisms induce isos and epis:



Basic constructions with NNOs

$$\begin{array}{ccccccccc}
* & \longmapsto & 0 & \longmapsto & 1 & \longmapsto & 2 & & m \\
& \searrow & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
& & (n\mapsto n) & \longmapsto & (n\mapsto n+1) & \longmapsto & (n\mapsto n+2) & & n\mapsto n+m
\end{array}$$

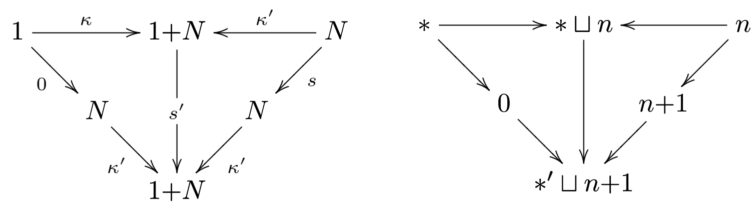
$$\begin{array}{ccccccccc}
* & \longmapsto & 0 & \longmapsto & 1 & \longmapsto & 2 & & m \\
& \searrow & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
& & (n\mapsto 0) & \longmapsto & (n\mapsto n) & \longmapsto & (n\mapsto 2n) & & n\mapsto mn
\end{array}$$

$$\begin{array}{ccccccccc}
* & \longmapsto & 0 & \longmapsto & 1 & \longmapsto & 2 & & m \\
& \searrow & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
& & (n\mapsto 1) & \longmapsto & (n\mapsto n) & \longmapsto & (n\mapsto n^2) & & n\mapsto n^m
\end{array}$$

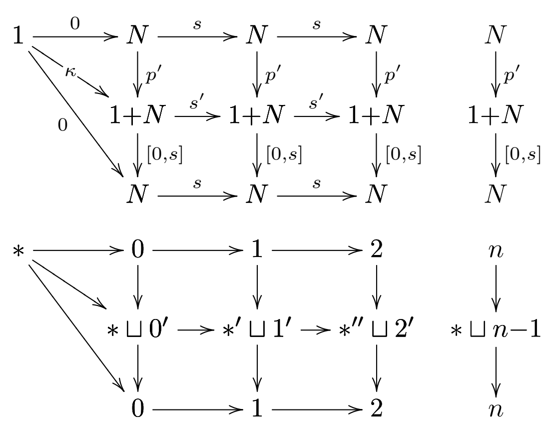
$$\begin{array}{ccccccccc}
* & \longmapsto & 0 & \longmapsto & 1 & \longmapsto & 2 & & m \\
& \searrow & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
& & (n\mapsto n) & \longmapsto & (n\mapsto n\dot{-}1) & \longmapsto & (n\mapsto n\dot{-}2) & & n\mapsto n\dot{-}m
\end{array}$$

NNOs: the morphism p'
Fact: in a topos with NNO the map $[0,s]:1{+}N \to N$ is an iso.

First we need to define the arrow $s':1{+}N \to 1{+}N$,
using a factorization through a coproduct.
Note that s' takes the '$*$' in '$* \sqcup n$' to 0, not to $*'$.
$s' := [0;\kappa', s;\kappa']$.



Now we can define $p':N \to 1+N$ by factoring (κ, s') through the NNO.
It is possible to show that p' and $[0,s]$ are inverses.



Define $n \mapsto n \dot{-} 1$ as $p';[0,\mathrm{id}]$,
The arrow $m \mapsto (n \dot{-} m)$ of the previous page