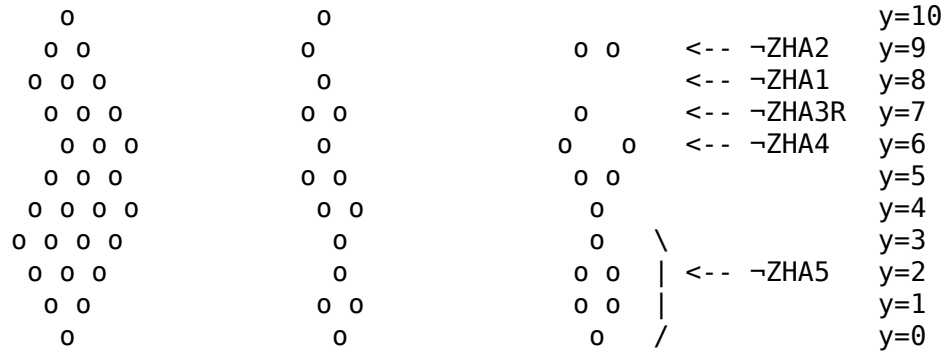




ZHAs - examples

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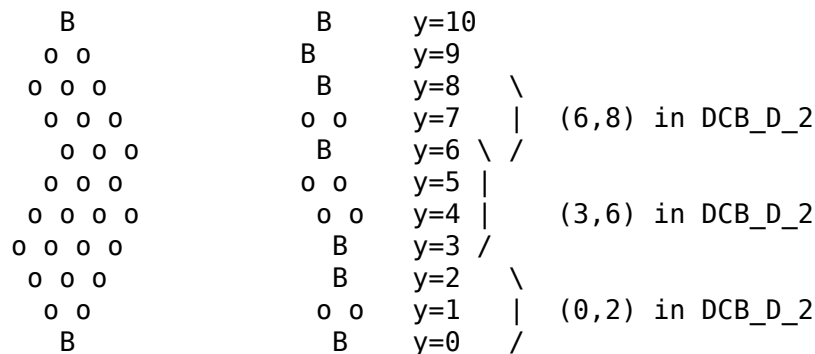
A ZHA (D_1) A ZHA (D_2) Not a ZHA (D_3)

A ZSet D is a ZHA iff it obeys these conditions:

- ZHA1: The non-empty lines of D are sequential
- ZHA2: D has a top element and a bottom element
- ZHA3L: The left wall of D can be traversed by black pawns moves
- ZHA3R: The right wall of D can be traversed by black pawns moves
- ZHA4: Each line of D is made of consecutive same-parity points
- ZHA5: All points in each wide region of D have the same parity

D_3 violates all the ZHA conditions...

- ZHA1: The non-empty lines of D_3 are not sequential - y=8 is a gap
- ZHA2: D_3 has more than one top element
- ZHA3R: this is not a black pawn's move (from y=7 to y=6)
- ZHA4: This line is not made of consecutive same-parity points (y=6)
- ZHA5: This wide region has points of opposite parities



A ZHA with one wide region

A ZHA with three wide regions

```

      B          B
    L R        B
  L o R        B
    L o R      L R
      L o R    B
    L o R      L R
  L o o R      L R
L o o R        B
  L o R        B
    L R        L R
      B        B
  
```

The left wall and the right walls of D_1 and D_2 ; the bottlenecks belong to both walls

```

      o          G          .
    o o        G          . . .
  G o o      G G        G . . .
    G o o    o          \ . G
      o o G   G o       . \ /
    o o o    G G       . . X G
  o o o G   G          G . / G
G o o G     o          G / G
  G o G     G G        G G
    G G     o
      o
  
```

The generators of D_1 and D_2 , and the inter-wall arrows of D_1

The intuitionistic implication:

$$\begin{aligned}
 \begin{array}{c} 0 \\ 0 \ 1 \ \rightarrow \ 1 \ 0 \\ 1 \quad I \quad 1 \end{array} &= \begin{array}{c} 0 \quad 0 \quad \text{int} \\ 0 \ 1 \ \rightarrow \ 1 \ 0 \\ 1 \quad M \quad 1 \end{array} \\
 &= \begin{array}{c} 1 \quad \text{int} \\ 1 \ 0 \\ 1 \end{array} \\
 &= \begin{array}{c} 0 \\ 1 \ 0 \\ 1 \end{array}
 \end{aligned}$$

P	Q	P&Q	P\ / Q	P->Q
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	1