Cálculo 2
PURO-UFF - 2015.2
Material para exercícios - Eduardo Ochs
Versão: 7/dez/2015
Links importantes:
http://angg.twu.net/2015.2-C2.html (página do curso)
http://angg.twu.net/2015.2-C2/2015.2-C2.pdf (quadros)
http://angg.twu.net/LATEX/2015-2-C2-material.pdf (lista, atualizada)
eduardoochs@gmail.com (meu e-mail)

Substituição:

$$g(h(x))\big|_{x=a}^{x=b} = \int_{x=a}^{x=b} g'(h(x))h'(x)\,dx$$

$$||$$

$$g(u))\big|_{u=h(a)}^{u=h(b)} = \int_{u=h(a)}^{u=h(b)} g'(u)\,du$$

$$\int_{x=a}^{x=b} f(g(x))g'(x)\,dx = \int_{u=g(a)}^{u=g(b)} f(u)\,du$$

Um exemplo de substituição:

$\int s^3c^3\,d\theta$	$\int \operatorname{sen}^3\theta\cos^3\theta\,d\theta$	$(\int_{\theta=\alpha}^{\theta=\beta} \square\,d\theta)$
$= \int s^3c^2c\,d\theta$	$= \int \operatorname{sen}^3\theta\cos^2\theta\cos\theta\,d\theta$	$(\int_{\theta=\alpha}^{\theta=\beta} \square\,d\theta)$
$= \int s^3(1-s^2)\frac{ds}{d\theta}\,d\theta$	$= \int \operatorname{sen}^3\theta(1-\operatorname{sen}^2\theta)\frac{d\operatorname{sen}\theta}{d\theta}\,d\theta$	$(\int_{\theta=\alpha}^{\theta=\beta} \square\,d\theta)$
$= \int (s^3-s^5)\frac{ds}{d\theta}\,d\theta$	$= \int (\operatorname{sen}^3\theta-\operatorname{sen}^5\theta)\frac{d\operatorname{sen}\theta}{d\theta}\,d\theta$	$(\int_{\theta=\alpha}^{\theta=\beta} \square\,d\theta)$
$= \int s^3-s^5\,ds$	$= \int s^3-s^5\,ds$	$(\int_{s=\operatorname{sen}\alpha}^{s=\operatorname{sen}\beta} \square\,ds)$
$= \frac{s^4}{4}-\frac{s^6}{6}$	$= \frac{s^4}{4}-\frac{s^6}{6}$	$(\square\big|_{s=\operatorname{sen}\alpha}^{s=\operatorname{sen}\beta})$
	$= \frac{\operatorname{sen}^4\theta}{4}-\frac{\operatorname{sen}^6\theta}{6}$	$(\square\big|_{\theta=\alpha}^{\theta=\beta})$

Substituição:

$$g(h(x))\Big|_{x=a}^{x=b} = \int_{x=a}^{x=b} g'(h(x))\,\frac{d\,h(x)}{dx}\,dx$$
$$||$$
$$g(u))\Big|_{u=h(a)}^{u=h(b)} = \int_{u=h(a)}^{u=h(b)} g'(u)\,du$$

Fórmulas:

$$\begin{aligned}&\int_{x=a}^{x=b} f(g(x))\,\frac{d\,g(x)}{dx}\,dx \\ &= \int_{x=a}^{x=b} f(u)\,\frac{du}{dx}\,dx \\ &= \int_{u=g(a)}^{u=g(b)} f(u)\,du\end{aligned}$$

$$\begin{aligned}&\int f(g(x))\,\frac{d\,g(x)}{dx}\,dx \\ &= \int f(u)\,\frac{du}{dx}\,dx && \left[\,u=g(x)\,\right] \\ &= \int f(u)\,du && \left[\,u=g(x)\,\right]\end{aligned}$$

Substituição inversa:

$$g(h(x))\Big|_{x=h^{-1}(\alpha)}^{x=h^{-1}(\beta)} = \int_{x=h^{-1}(\alpha)}^{x=h^{-1}(\beta)} g'(h(x))\,\frac{d\,h(x)}{dx}\,dx$$
$$||$$
$$g(u))\Big|_{u=h(h^{-1}(\alpha))}^{u=h(h^{-1}(\beta))} = \int_{u=h(h^{-1}(\alpha))}^{u=h(h^{-1}(\beta))} g'(u)\,du$$
$$||$$
$$g(u))\Big|_{u=\alpha}^{u=\beta} = \int_{u=\alpha}^{u=\beta} g'(u)\,du$$

Fórmulas:

$$\begin{aligned}&\int_{u=\alpha}^{u=\beta} f(u)\,du \\ &= \int_{x=g^{-1}(\alpha)}^{x=g^{-1}(\beta)} f(u)\,\frac{du}{dx}\,dx \\ &= \int_{x=g^{-1}(\alpha)}^{x=g^{-1}(\beta)} f(g(x))\,\frac{d\,g(x)}{dx}\,dx\end{aligned}$$

$$\begin{aligned}&\int f(u)\,du \\ &= \int f(u)\,\frac{du}{dx}\,dx && \left[\begin{smallmatrix}u=g(x)\\ x=g^{-1}(u)\end{smallmatrix}\right] \\ &= \int f(g(x))\,\frac{d\,g(x)}{dx}\,dx && \left[\,x=g^{-1}(u)\,\right]\end{aligned}$$

Substituição trigonométrica:

$$\begin{aligned}&\int_{s=a}^{s=b} F(s,\sqrt{1-s^2})\,ds \\ &= \int_{\theta=\mathrm{arcsen}\,a}^{\theta=\mathrm{arcsen}\,b} F(\mathrm{sen}\,\theta,\sqrt{1-\mathrm{sen}^2\,\theta})\,\frac{d\,\mathrm{sen}\,\theta}{d\theta}\,d\theta \\ &= \int_{\theta=\mathrm{arcsen}\,a}^{\theta=\mathrm{arcsen}\,b} F(\mathrm{sen}\,\theta,\cos\theta)\cos\theta\,d\theta\end{aligned}$$

$$\begin{aligned}&\int F(s,\sqrt{1-s^2})\,ds \\ &= \int F(s,\sqrt{1-s^2})\,\frac{ds}{d\theta}\,d\theta && \left[\begin{smallmatrix}s=\mathrm{sen}\,\theta\\ \theta=\mathrm{arcsen}\,\theta\end{smallmatrix}\right] \\ &= \int F(s,c)c\,d\theta && \left[\begin{smallmatrix}s=\mathrm{sen}\,\theta\\ c=\cos\theta\\ \theta=\mathrm{arcsen}\,\theta\end{smallmatrix}\right]\end{aligned}$$

$$\begin{aligned}&\int_{z=a}^{z=b} F(z,\sqrt{z^2-1})\,dz \\ &= \int_{\theta=\mathrm{arcsec}\,a}^{\theta=\mathrm{arcsec}\,b} F(\sec\theta,\sqrt{\sec^2\theta-1})\,\frac{d\sec\theta}{d\theta}\,d\theta \\ &= \int_{\theta=\mathrm{arcsec}\,a}^{\theta=\mathrm{arcsec}\,b} F(\sec\theta,\tan\theta)\sec\theta\tan\theta\,d\theta\end{aligned}$$

$$\begin{aligned}&\int F(z,\sqrt{z^2-1})\,dz \\ &= \int F(z,\sqrt{z^2-1})\,\frac{dz}{d\theta}\,d\theta && \left[\begin{smallmatrix}z=\sec\theta\\ \theta=\mathrm{arcsec}\,z\end{smallmatrix}\right] \\ &= \int F(z,t)zt\,d\theta && \left[\begin{smallmatrix}z=\sec\theta\\ \theta=\mathrm{arcsec}\,z\\ t=\tan\theta\end{smallmatrix}\right]\end{aligned}$$

$$\begin{aligned}&\int_{t=a}^{t=b} F(t,\sqrt{1+t^2})\,dt \\ &= \int_{\theta=\arctan a}^{\theta=\arctan b} F(\tan\theta,\sqrt{1+\tan^2\theta})\,\frac{d\tan\theta}{d\theta}\,d\theta \\ &= \int_{\theta=\arctan a}^{\theta=\arctan b} F(\tan\theta,\sec\theta)\sec^2\theta\,d\theta\end{aligned}$$

$$\begin{aligned}&\int F(t,\sqrt{1+t^2})\,dt \\ &= \int F(t,\sqrt{1+t^2})\,\frac{dt}{d\theta}\,d\theta && \left[\begin{smallmatrix}t=\tan\theta\\ \theta=\arctan t\end{smallmatrix}\right] \\ &= \int F(t,z)z^2\,d\theta && \left[\begin{smallmatrix}t=\tan\theta\\ \theta=\arctan t\\ z=\sec\theta\end{smallmatrix}\right]\end{aligned}$$

Algumas fórmulas:

Integração por partes:

$\int_{x=a}^{x=b} f'(x)g(x)\,dx = f(x)g(x)\Big|_{x=a}^{x=b} - \int_{x=a}^{x=b} f(x)g'(x)\,dx$

Integrais de $(\operatorname{sen}\theta)^m(\cos\theta)^n$ com um expoente ímpar ($s = \operatorname{sen}\theta$, $c = \cos\theta$):

$$\int s^n c^{2k+1} d\theta = \int s^n c^{2k} \cdot c\,d\theta = \begin{bmatrix} \scriptstyle \operatorname{sen}\theta = s \\ \scriptstyle \cos^2\theta = 1-s^2 \\ \scriptstyle \cos\theta\,d\theta = ds \\ \scriptstyle \theta = \operatorname{arcsen} s \end{bmatrix} \int s^n(1-s^2)^k\,ds$$

$$\int c^n s^{2k+1} d\theta = \int c^n s^{2k} \cdot s\,d\theta = \begin{bmatrix} \scriptstyle \cos\theta = c \\ \scriptstyle \operatorname{sen}^2\theta = 1-c^2 \\ \scriptstyle -\operatorname{sen}\theta\,d\theta = dc \\ \scriptstyle \theta = \arccos s \end{bmatrix} -\int c^n(1-c^2)^k\,dc$$

Substituição trigonométrica:

$$\int F(s, \sqrt{1-s^2})\,ds = \begin{bmatrix} \scriptstyle s = \operatorname{sen}\theta \\ \scriptstyle \sqrt{1-s^2} = \cos\theta \\ \scriptstyle ds = \cos\theta\,d\theta \\ \scriptstyle \theta = \operatorname{arcsen} s \end{bmatrix} \int F(\operatorname{sen}\theta, \cos\theta)\cos\theta\,d\theta$$

$$\int F(t, \sqrt{1+t^2})\,dt = \begin{bmatrix} \scriptstyle t = \tan\theta \\ \scriptstyle \sqrt{1+t^2} = \sec\theta \\ \scriptstyle dt = \sec^2\theta\,d\theta \\ \scriptstyle \theta = \arctan t \end{bmatrix} \int F(\tan\theta, \sec\theta)\sec^2\theta\,d\theta$$

$$\int F(z, \sqrt{z^2-1})\,dz = \begin{bmatrix} \scriptstyle z = \sec\theta \\ \scriptstyle \sqrt{z^2-1} = \tan\theta \\ \scriptstyle dz = \tan\theta\sec\theta\,d\theta \\ \scriptstyle \theta = \operatorname{arcsec} z \end{bmatrix} \int F(\sec\theta, \tan\theta)\tan\theta\sec\theta\,d\theta$$

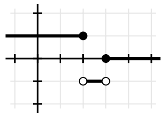
$$\int \sqrt{1-x^2}\,dx = \frac{\operatorname{arcsen} x}{2} + \frac{x\sqrt{1-x^2}}{2}$$

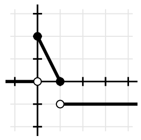
Método de Heaviside:

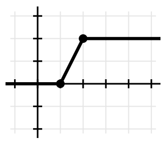
Se $f(x) = \frac{\alpha}{x-a} + \frac{\beta}{x-b} + \frac{\gamma}{x-c} = \frac{p(x)}{(x-a)(x-b)(x-c)}$,

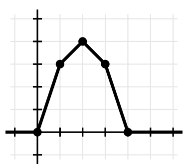
então $\lim_{x\to a} f(x)(x-a) = \alpha = \frac{p(a)}{(a-b)(a-c)}$.

Funções usadas nas aulas de 30/nov e 2/dez/2015:

$f_1(x) =$ 

$f_2(x) =$ 

$f_3(x) =$ 

$f_4(x) =$ 

$f_6(x) =$