Geometria Analítica
PURO-UFF - 2017.1
P2 - 17/jul/2017 - Eduardo Ochs
Respostas sem justificativas não serão aceitas.
Diagramas muito ambíguos *serão* interpretados errado.
Proibido usar quaisquer aparelhos eletrônicos.

Lembre que uma equação de cônica é uma equação da forma $ax^2+bxy+cy^2+dy+ey+f=0$; $4+(x+y)(x-y)=5y$ não é uma equação de cônica mas é equivalente a uma: $x^2-y^2-5y+4=0$. E o truque pra gente se livrar das duas raízes quadradas em $\sqrt{A}+\sqrt{B}=C$ ou $\sqrt{A}-\sqrt{B}=C$ é:

$$\begin{array}{lcl} \sqrt{A}+\sqrt{B}=C & \Rightarrow & C^2(C^2-2(A+B))+(A-B)^2=0 \\ \sqrt{A}-\sqrt{B}=C & \Rightarrow & C^2(C^2-2(A+B))+(A-B)^2=0 \end{array}$$

1) **(Total: 4.5)** Faça esboços das cônicas com as equações abaixo. Algumas delas são degeneradas. Em todos os itens abaixo considere que $u=y-x/2$ e $v=y+x/2$ — ou que u e v são *abreviações* para $y-x/2$ e $y+x/2$.

a) (0.1 pts)	$x^2+y^2=1$	k) (0.5 pts)	$u(u-1)=0$
b) (0.1 pts)	$x^2+y^2=4$	l) (0.5 pts)	$v(v-1)=0$
c) (0.1 pts)	$x^2+y^2=0$	m) (0.2 pts)	$u^2+v^2=0$
d) (0.1 pts)	$xy=1$	n) (0.3 pts)	$u^2+v^2=1$
e) (0.1 pts)	$xy=4$	o) (0.2 pts)	$uv=0$
f) (0.1 pts)	$xy=0$	p) (0.4 pts)	$uv=1$
g) (0.1 pts)	$xy=-1$	q) (0.4 pts)	$uv=-1$
h) (0.1 pts)	$x^2=y$	r) (0.5 pts)	$u^2=v$
i) (0.1 pts)	$x=y^2$	s) (0.5 pts)	$u=v^2$
j) (0.1 pts)	$x^2=y^2$		

2) **(Total: 1.5)** Seja $S=\{\,(x,y)\in\mathbb{R}^2 \mid y-x/2=(y+x/2)^2\,\}$ (veja o item s da questão anterior). Dê as coordenadas (x,y) de cinco pontos de S.
Dica: confira os seus resultados!!!

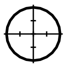
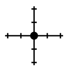
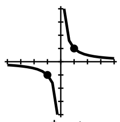
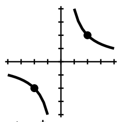
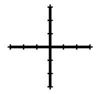
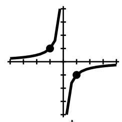
3) **(Total: 1.5)** Sejam $F=(-1,0)$, $F'=(1,0)$, e

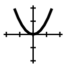
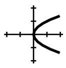
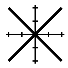
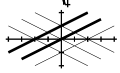
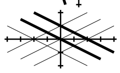
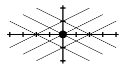
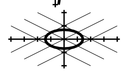
$$\begin{array}{rcl} S &=& \{\,(x,y)\in\mathbb{R}^2 \mid d((x,y),F)+d((x,y),F')=4\,\} \\ &=& \{\,(x,y)\in\mathbb{R}^2 \mid \sqrt{(x+1)^2+y^2}+\sqrt{(x-1)^2+y^2}=4\,\}, \\ S' &=& \{\,(x,y)\in\mathbb{R}^2 \mid ax^2+bx+c+dxy+ey+fy^2=0\,\}. \end{array}$$

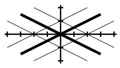
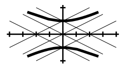
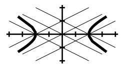
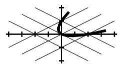
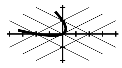
a) **(1.0 pts)** Encontre $a,b,c,d,e,f\in\mathbb{R}$ que façam com que $S=S'$.
b) **(0.5 pts)** Dê as coordenadas de 4 pontos da elipse S.

4) **(Total: 2.5)** Sejam $A=(0,0,2)$, $B=(0,1,0)$, $C=(1,0,0)$, $D=(1,1,0)$, π o plano que contém A,B,C, π' o plano paralelo a π que contém o ponto D.
a) **(0.2 pts)** Dê a equação do plano π.
b) **(0.3 pts)** Dê a equação do plano π'.
c) **(0.5 pts)** Calcule $d(\pi,\pi')$.
d) **(1.5 pts)** Dê as coordenadas do ponto de π' mais próximo de A.

Gabarito (versão revisada)

1a)  1b)  1c)  1d)  1e)  1f)  1g) 

1h)  1i)  1j)  1k)  1l)  1m)  1n) 

1o)  1p)  1q)  1r)  1s) 

2)

u	v	x	y	
4	-2	-6	1	(0.5 pts)
1	-1	-2	0	(0.2 pts)
0	0	0	0	(0.1 pts)
1	1	0	1	(0.2 pts)
4	2	-2	3	(0.5 pts)

3a) Sejam

$$\begin{aligned} A &= (x+1)^2 + y^2 = x^2 + 2x + 1 + y^2, \\ B &= (x-1)^2 + y^2 = x^2 - 2x + 1 + y^2, \\ C &= 4. \end{aligned}$$

Então

$$\begin{aligned} A + B &= 2x^2 + 2y^2 + 2, \\ A - B &= 4x, \\ C^2 - 2(A+B) &= 16 - 4x^2 - 4y^2 - 4 = 12 - 4x^2 - 4y^2, \\ C^2(C^2 - 2(A+B)) &= 192 - 64x^2 - 64y^2, \\ (A-B)^2 &= 16x^2, \\ C^2(C^2 - 2(A+B)) + (A-B)^2 &= 192 - 48x^2 - 64y^2 \\ &= 192(1 - \tfrac{1}{4}x^2 - \tfrac{1}{3}y^2), \end{aligned}$$

$$\begin{aligned} S' &= \{\, (x,y) \in \mathbb{R}^2 \mid -48x^2 + 0x + 192 + 0xy + 0y - 64y^2 = 0 \,\} \\ &= \{\, (x,y) \in \mathbb{R}^2 \mid \tfrac{1}{4}x^2 + 0x - 1 + 0xy + 0y + \tfrac{1}{3}y^2 = 0 \,\}. \end{aligned}$$

3b) $(0, \sqrt{3})$,
$(-2, 0)$, $(2, 0)$,
$(0, -\sqrt{3})$.

4a) $\pi = \{\, (x,y,z) \in \mathbb{R}^3 \mid x + y + \frac{z}{2} = 1 \,\}$
4b) $\pi' = \{\, (x,y,z) \in \mathbb{R}^3 \mid x + y + \frac{z}{2} = 2 \,\}$
4c) Sejam $\vec{u} = \overrightarrow{AB} = \overrightarrow{(0,1,-2)}$, $\vec{v} = \overrightarrow{AC} = \overrightarrow{(1,0,-2)}$, $\vec{w} = \overrightarrow{AD} = \overrightarrow{(1,1,-2)}$. Então

$$\begin{aligned} \vec{u} \times \vec{v} &= \overrightarrow{(-2,-2,-1)}, \\ \mathsf{área}(\vec{u},\vec{v}) &= \sqrt{2^2 + 2^2 + 1^2} = 3, \\ |[\vec{u},\vec{v},\vec{w}]| &= \begin{vmatrix} 0 & 1 & -2 \\ 1 & 0 & -2 \\ 1 & 1 & -2 \end{vmatrix} = -2, \\ d(\pi,\pi') &= \left| \frac{|[\vec{u},\vec{v},\vec{w}]|}{\mathsf{área}(\vec{u},\vec{v})} \right| = |-2/3| = 2/3. \end{aligned}$$

4d) A reta $r = \{\, A + t\overrightarrow{(2,2,1)} \mid t \in \mathbb{R} \,\} = \{\, (2t, 2t, 2+t) \mid t \in \mathbb{R} \,\}$ passa por A e é ortogonal a π. Ela intersecta π' quando $2t + 2t + \frac{2+t}{2} = 2$, i.e., quando $\frac{9}{2}t = 1$ e quando $t = \frac{2}{9}$, o que corresponde ao ponto $P = (\frac{4}{9}, \frac{4}{9}, \frac{20}{9})$.