

Cálculo 2
 PURO-UFF - 2018.2
 P1 - 12/nov/2018 - Eduardo Ochs
 Respostas sem justificativas não serão aceitas.
 Proibido usar quaisquer aparelhos eletrônicos.

1) **(Total: 2.0)** Calcule

$$\int (\operatorname{sen} x)^4 (\cos x)^2 dx.$$

2) **(Total: 2.0)** Calcule

$$\int \frac{x^2}{\sqrt{4x^2 - 9}} dx.$$

3) **(Total: 2.0)** Calcule

$$\int \frac{x^3}{x^2 + 7x + 12} dx.$$

4) **(Total: 2.0)** Calcule

$$\int \frac{x^4}{(x - 3)^2} dx.$$

5) **(Total: 2.0)** Calcule por integração por partes:

a) **(1.0 pts)** $\int 1 \cdot \ln x dx$,

b) **(1.0 pts)** $\int x \cdot \ln x dx$.

Algumas definições, fórmulas e substituições:

$$\begin{array}{llll} c = \cos \theta & c^2 + s^2 = 1 & \frac{ds}{d\theta} = c & E = c + is \\ s = \operatorname{sen} \theta & z^2 = t^2 + 1 & \frac{dc}{d\theta} = -s & c = \frac{E+E^{-1}}{2} \\ t = \tan \theta & \sqrt{1-s^2} = c & \frac{dt}{d\theta} = z^2 & s = \frac{E-E^{-1}}{2i} \\ z = \sec \theta & \sqrt{t^2+1} = z & \frac{dz}{d\theta} = zt & e^{ik\theta} + e^{-ik\theta} = 2 \cos k\theta \\ E = e^{i\theta} & \sqrt{z^2-1} = t & & e^{ik\theta} - e^{-ik\theta} = 2i \operatorname{sen} k\theta \end{array}$$

Gabarito

$$\begin{aligned}
1) \quad (\operatorname{sen} \theta)^4 &= \left(\frac{E-E^{-1}}{2i} \right)^4 = \frac{1}{16}(E^4 - 4E^2 + 6 - 4E^{-2} + E^{-4}) \\
(\cos \theta)^2 &= \left(\frac{E+E^{-1}}{2} \right)^2 = \frac{1}{4}(E^2 + 2 + E^{-2}) \\
(\operatorname{sen} \theta)^4 (\cos \theta)^2 &= \frac{1}{64} \left(\begin{array}{cccccc} E^6 & -4E^4 & +6E^2 & -4 & +E^{-2} & \\ & +2E^4 & -8E^2 & +12 & -8E^{-2} & +2E^{-4} \\ & & +E^2 & -4 & +6E^{-2} & -4E^{-4} & +E^{-6} \end{array} \right) \\
&= \frac{1}{64}(E^6 - 2E^4 - E^2 + 4 - E^{-2} - 2E^{-4} + E^{-6}) \\
&= \frac{1}{64}((E^6 + E^{-6}) - 2(E^4 + E^{-4}) - (E^2 + E^{-2}) + 4) \\
&= \frac{1}{64}(2 \cos 6\theta - 4 \cos 4\theta - 2 \cos 2\theta + 4) \\
\int (\operatorname{sen} \theta)^4 (\cos \theta)^2 d\theta &= \frac{2}{64 \cdot 6} \operatorname{sen} 6\theta + \frac{4}{64 \cdot 4} \operatorname{sen} 4\theta - \frac{2}{64 \cdot 2} \operatorname{sen} 2\theta + \frac{4}{64} \theta
\end{aligned}$$

$$\begin{aligned}
2) \quad \int \frac{x^2}{\sqrt{4x^2 - 9}} dx &= \int \frac{x^2}{3\sqrt{\frac{4}{9}x^2 - 1}} dx \\
&= \int \frac{x^2}{3\sqrt{\left(\frac{2}{3}x\right)^2 - 1}} dx \quad \left[\begin{array}{l} \frac{2}{3}x = z \\ x = \frac{3}{2}z \\ dx = \frac{3}{2}dz \end{array} \right] \\
&= \int \frac{\left(\frac{3}{2}z\right)^2}{3\sqrt{z^2 - 1}} \frac{3}{2} dz \\
&= \frac{9}{8} \int \frac{z^2}{\sqrt{z^2 - 1}} dz \quad \left[\begin{array}{l} z = \sec \theta = \frac{1}{c} \\ \sqrt{z^2 - 1} = \tan \theta = \frac{s}{c} \\ dz = z d\theta = \frac{s}{c^2} d\theta \end{array} \right] \\
&= \frac{9}{8} \int \frac{c^{-2}}{sc^{-1}} sc^{-2} d\theta \\
&= \frac{9}{8} \int c^{-3} d\theta \\
&= \frac{9}{8} \int c^{-4} c d\theta \quad \left[\begin{array}{l} c^2 = 1 - s^2 \\ c d\theta = ds \end{array} \right] \\
&= \frac{9}{8} \int \frac{1}{(1 - s^2)^2} ds \\
&= \frac{9}{8} \int \frac{1}{(s^2 - 1)^2} ds \\
&= \frac{9}{8} \int \frac{1}{(s+1)^2(s-1)^2} ds \\
&= \frac{9}{8} \int \frac{1}{4} \left(\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s-1} + \frac{1}{(s-1)^2} \right) ds \\
&= \frac{9}{32} \left(\ln|s+1| - \frac{1}{s+1} + \ln|s-1| - \frac{1}{s-1} \right) \\
&= \frac{9}{32} \left(\ln\left|\frac{1}{z} + 1\right| - \frac{1}{\frac{1}{z} + 1} + \ln\left|\frac{1}{z} - 1\right| - \frac{1}{\frac{1}{z} - 1} \right) \\
&= \frac{9}{32} \left(\ln\left|\frac{3}{2x} + 1\right| - \frac{1}{\frac{3}{2x} + 1} + \ln\left|\frac{3}{2x} - 1\right| - \frac{1}{\frac{3}{2x} - 1} \right)
\end{aligned}$$

$$\begin{aligned}
3) \int \frac{x^3}{x^2 + 7x + 12} dx &= \int x - 7 + \frac{37x + 84}{(x+3)(x+4)} dx \\
&= \int x - 7 - \frac{27}{x+3} + \frac{64}{x+4} dx \\
&= \frac{x^2}{2} - 7x - 27 \ln|x+3| + 64 \ln|x+4|
\end{aligned}$$

$$\begin{aligned}
4) \int \frac{x^4}{(x-3)^2} dx &\left[\begin{array}{l} u=x-3 \\ x=u+3 \\ dx=du \end{array} \right] \\
&= \int \frac{(u+3)^4}{u^2} du \\
&= \int \frac{u^4 + 4 \cdot u^3 \cdot 3 + 6 \cdot u^2 \cdot 9 + 4 \cdot u \cdot 27 + 81}{u^2} du \\
&= \int \frac{u^4 + 12u^3 + 54u^2 + 108u + 81}{u^2} du \\
&= \int u^2 + 12u + 54 + \frac{108}{u} + \frac{81}{u^2} du \\
&= \frac{u^3}{3} + 6u^2 + 54u + 108 \ln|u| - \frac{81}{u} \\
&= \frac{(x-3)^3}{3} + 6(x-3)^2 + 54(x-3) + 108 \ln|x-3| - \frac{81}{x-3}
\end{aligned}$$

$$5a) \int \underbrace{1}_{f'} \cdot \underbrace{\ln x}_g dx = \underbrace{x}_f \cdot \underbrace{\ln x}_g - \int \underbrace{x}_f \cdot \underbrace{\frac{1}{x}}_{g'} dx = x \ln x - \int 1 dx = x \ln x - x$$

$$\begin{aligned}
5b) \int \underbrace{x}_{f'} \cdot \underbrace{\ln x}_g dx &= \underbrace{\frac{x^2}{2}}_f \cdot \underbrace{\ln x}_g - \int \underbrace{\frac{x^2}{2}}_f \cdot \underbrace{\frac{1}{x}}_{g'} dx = \frac{1}{2}x^2 \ln x - \frac{1}{2} \int x dx \\
&= \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2
\end{aligned}$$