

“Set comprehensions”

Notação explícita, com geradores, filtros,

e um “;” separando os geradores e filtros da expressão final:

$$\begin{aligned}
 \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{10a}_{\text{expr}} &= \{10, 20, 30, 40\} \\
 \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{a}_{\text{expr}} &= \{1, 2, 3, 4\} \\
 \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{a \geq 3}_{\text{filt}}; \underbrace{a}_{\text{expr}} &= \{3, 4\} \\
 \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{a \geq 3}_{\text{filt}}; \underbrace{10a}_{\text{expr}} &= \{30, 40\} \\
 \underbrace{\{a \in \{10, 20\}\}}_{\text{ger}}; \underbrace{b \in \{3, 4\}}_{\text{ger}}; \underbrace{a + b}_{\text{expr}} &= \{13, 14, 23, 24\} \\
 \underbrace{\{a \in \{1, 2\}\}}_{\text{ger}}; \underbrace{b \in \{3, 4\}}_{\text{ger}}; \underbrace{(a, b)}_{\text{expr}} &= \{(1, 3), (1, 4), (2, 3), (2, 4)\}
 \end{aligned}$$

Notações convencionais, com “|” ao invés de “;”:

Primeiro tipo — expressão final, “|”, geradores e filtros:

$$\begin{aligned}
 \{10a \mid a \in \{1, 2, 3, 4\}\} &= \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{10a}_{\text{expr}} \\
 \{10a \mid a \in \{1, 2, 3, 4\}, a \geq 3\} &= \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{a \geq 3}_{\text{filt}}; \underbrace{10a}_{\text{expr}} \\
 \{a \mid a \in \{1, 2, 3, 4\}\} &= \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{a}_{\text{expr}}
 \end{aligned}$$

O segundo tipo — gerador, “|”, filtros —

pode ser convertido para o primeiro...

o truque é fazer a expressão final ser a variável do gerador:

$$\begin{aligned}
 \{a \in \{1, 2, 3, 4\} \mid a \geq 3\} &= \\
 \{a \mid a \in \{1, 2, 3, 4\}, a \geq 3\} &= \underbrace{\{a \in \{1, 2, 3, 4\}\}}_{\text{ger}}; \underbrace{a \geq 3}_{\text{filt}}; \underbrace{a}_{\text{expr}}
 \end{aligned}$$

O que distingue as duas notações “{...|...}” é

se o que vem antes da “|” é ou não um gerador.

Observações:

$$\{\text{gerador} \mid \text{filtros}\} = \{\text{gerador, filtros}; \underbrace{\text{variável do gerador}}_{\text{expr}}\}$$

$$\{\text{expr} \mid \text{geradores e filtros}\} = \{\text{geradores e filtros}; \text{expr}\}$$

As notações “{...|...}” são padrão e são usadas em muitos livros de matemática.

A notação “{...;...}” é bem rara; eu aprendi ela em artigos sobre linguagens de programação, e resolvi apresentar ela aqui porque acho que ela ajuda a explicar as duas notações “{...|...}”.

“Set comprehensions”: como calcular usando tabelas

Alguns exemplos:

Se $A := \{x \in \{1, 2\}; (x, 3 - x)\}$

então $A = \{(1, 2), (2, 1)\}$:

x	$(x, 3-x)$
1	(1,2)
2	(2,1)

Se $I := \{x \in \{1, 2, 3\}, y \in \{3, 4\}, x + y < 6; (x, y)\}$

então $I = \{(1, 3), (1, 4), (1, 5)\}$:

x	y	$x+y < 6$	(x, y)
1	3	V	(1,3)
1	4	V	(1,4)
2	3	V	(2,3)
2	4	F	
3	3	F	
3	4	F	

Se $D := \{(x, 2x) \mid x \in \{0, 1, 2, 3\}\}$

então $D = \{x \in \{0, 1, 2, 3\}; (x, 2x)\}$,

$D = \{(0, 0), (1, 2), (2, 4), (3, 6)\}$:

x	$(x, 2x)$
0	(0,0)
1	(1,2)
2	(2,4)
3	(3,6)

Se $P := \{(x, y) \in \{1, 2, 3\}^2 \mid x \geq y\}$

então $P = \{(x, y) \in \{1, 2, 3\}^2, x \geq y; (x, y)\}$,

$P = \{(1, 1), (2, 1), (2, 2), (3, 1), (3, 2), (3, 3)\}$:

(x, y)	x	y	$x \geq y$	(x, y)
(1,1)	1	1	V	(1,1)
(1,2)	1	2	F	
(1,3)	1	3	F	
(2,1)	2	1	V	(2,1)
(2,2)	2	2	V	(2,2)
(2,3)	2	3	F	
(3,1)	3	1	V	(3,1)
(3,2)	3	2	V	(3,2)
(3,3)	3	3	V	(3,3)

Obs: os exemplos acima correspondem aos exercícios 2A, 2I, 3D e 5P das próximas páginas.

Exercícios de “set comprehensions”

1) Represente graficamente:

$$A := \{(1, 4), (2, 4), (1, 3)\}$$

$$B := \{(1, 3), (1, 4), (2, 4)\}$$

$$C := \{(1, 3), (1, 4), (2, 4), (2, 4)\}$$

$$D := \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

$$E := \{(0, 3), (1, 2), (2, 1), (3, 0)\}$$

2) Calcule e represente graficamente:

$$A := \{x \in \{1, 2\}; (x, 3 - x)\}$$

$$B := \{x \in \{1, 2, 3\}; (x, 3 - x)\}$$

$$C := \{x \in \{0, 1, 2, 3\}; (x, 3 - x)\}$$

$$D := \{x \in \{0, 0.5, 1, \dots, 3\}; (x, 3 - x)\}$$

$$E := \{x \in \{1, 2, 3\}, y \in \{3, 4\}; (x, y)\}$$

$$F := \{x \in \{3, 4\}, y \in \{1, 2, 3\}; (x, y)\}$$

$$G := \{x \in \{3, 4\}, y \in \{1, 2, 3\}; (y, x)\}$$

$$H := \{x \in \{3, 4\}, y \in \{1, 2, 3\}; (x, 2)\}$$

$$I := \{x \in \{1, 2, 3\}, y \in \{3, 4\}, x + y < 6; (x, y)\}$$

$$J := \{x \in \{1, 2, 3\}, y \in \{3, 4\}, x + y > 4; (x, y)\}$$

$$K := \{x \in \{1, 2, 3, 4\}, y \in \{1, 2, 3, 4\}; (x, y)\}$$

$$L := \{x, y \in \{0, 1, 2, 3, 4\}; (x, y)\}$$

$$M := \{x, y \in \{0, 1, 2, 3, 4\}, y = 3; (x, y)\}$$

$$N := \{x, y \in \{0, 1, 2, 3, 4\}, x = 2; (x, y)\}$$

$$O := \{x, y \in \{0, 1, 2, 3, 4\}, x + y = 3; (x, y)\}$$

$$P := \{x, y \in \{0, 1, 2, 3, 4\}, y = x; (x, y)\}$$

$$Q := \{x, y \in \{0, 1, 2, 3, 4\}, y = x + 1; (x, y)\}$$

$$R := \{x, y \in \{0, 1, 2, 3, 4\}, y = 2x; (x, y)\}$$

$$S := \{x, y \in \{0, 1, 2, 3, 4\}, y = 2x + 1; (x, y)\}$$

3) Calcule e represente graficamente:

$$A := \{(x, 0) \mid x \in \{0, 1, 2, 3\}\}$$

$$B := \{(x, x/2) \mid x \in \{0, 1, 2, 3\}\}$$

$$C := \{(x, x) \mid x \in \{0, 1, 2, 3\}\}$$

$$D := \{(x, 2x) \mid x \in \{0, 1, 2, 3\}\}$$

$$E := \{(x, 1) \mid x \in \{0, 1, 2, 3\}\}$$

$$F := \{(x, 1 + x/2) \mid x \in \{0, 1, 2, 3\}\}$$

$$G := \{(x, 1 + x) \mid x \in \{0, 1, 2, 3\}\}$$

$$H := \{(x, 1 + 2x) \mid x \in \{0, 1, 2, 3\}\}$$

$$I := \{(x, 2) \mid x \in \{0, 1, 2, 3\}\}$$

$$J := \{(x, 2 + x/2) \mid x \in \{0, 1, 2, 3\}\}$$

$$K := \{(x, 2 + x) \mid x \in \{0, 1, 2, 3\}\}$$

$$L := \{(x, 2 + 2x) \mid x \in \{0, 1, 2, 3\}\}$$

$$M := \{(x, 2) \mid x \in \{0, 1, 2, 3\}\}$$

$$N := \{(x, 2 - x/2) \mid x \in \{0, 1, 2, 3\}\}$$

$$O := \{(x, 2 - x) \mid x \in \{0, 1, 2, 3\}\}$$

$$P := \{(x, 2 - 2x) \mid x \in \{0, 1, 2, 3\}\}$$

Produto cartesiano de conjuntos

$$A \times B := \{a \in A, b \in B; (a, b)\}$$

$$\text{Exemplo: } \{1, 2\} \times \{3, 4\} = \{(1, 3), (1, 4), (2, 3), (2, 4)\}.$$

$$\text{Uma notação: } A^2 = A \times A.$$

$$\text{Exemplo: } \{3, 4\}^2 = \{3, 4\} \times \{3, 4\} = \{(3, 3), (3, 4), (4, 3), (4, 4)\}.$$

Sejam:

$$A = \{1, 2, 4\},$$

$$B = \{2, 3\},$$

$$C = \{2, 3, 4\}.$$

Exercícios

4) Calcule e represente graficamente:

$$\text{a) } A \times A \quad \text{d) } B \times A \quad \text{g) } C \times A$$

$$\text{b) } A \times B \quad \text{e) } B \times B \quad \text{h) } C \times B$$

$$\text{c) } A \times C \quad \text{f) } B \times C \quad \text{i) } C \times C$$

5) Calcule e represente graficamente:

$$A := \{x, y \in \{0, 1, 2, 3\}; (x, y)\}$$

$$B := \{x, y \in \{0, 1, 2, 3\}, y = 2; (x, y)\}$$

$$C := \{x, y \in \{0, 1, 2, 3\}, x = 1; (x, y)\}$$

$$D := \{x, y \in \{0, 1, 2, 3\}, y = x; (x, y)\}$$

$$E := \{x, y \in \{0, 1, 2, 3, 4\}, y = 2x; (x, y)\}$$

$$F := \{(x, y) \in \{0, 1, 2, 3, 4\}^2, y = 2x; (x, y)\}$$

$$G := \{(x, y) \in \{0, 1, 2, 3, 4\}^2, y = x; (x, y)\}$$

$$H := \{(x, y) \in \{0, 1, 2, 3, 4\}^2, y = x/2; (x, y)\}$$

$$I := \{(x, y) \in \{0, 1, 2, 3, 4\}^2, y = x/2 + 1; (x, y)\}$$

$$J := \{(x, y) \in \{0, 1, 2, 3, 4\}^2 \mid y = 2x\}$$

$$K := \{(x, y) \in \{0, 1, 2, 3, 4\}^2 \mid y = x\}$$

$$L := \{(x, y) \in \{0, 1, 2, 3, 4\}^2 \mid y = x/2\}$$

$$M := \{(x, y) \in \{0, 1, 2, 3, 4\}^2 \mid y = x/2 + 1\}$$

$$N := \{(x, y) \in \{1, 2, 3\}^2 \mid 0x + 0y = 0\}$$

$$O := \{(x, y) \in \{1, 2, 3\}^2 \mid 0x + 0y = 2\}$$

$$P := \{(x, y) \in \{1, 2, 3\}^2 \mid x \geq y\}$$

6) Represente graficamente:

$$J' := \{(x, y) \in \mathbb{R}^2 \mid y = 2x\}$$

$$K' := \{(x, y) \in \mathbb{R}^2 \mid y = x\}$$

$$L' := \{(x, y) \in \mathbb{R}^2 \mid y = x/2\}$$

$$M' := \{(x, y) \in \mathbb{R}^2 \mid y = x/2 + 1\}$$

$$N' := \{(x, y) \in \mathbb{R}^2 \mid 0x + 0y = 0\}$$

$$O' := \{(x, y) \in \mathbb{R}^2 \mid 0x + 0y = 2\}$$

$$P' := \{(x, y) \in \mathbb{R}^2 \mid x \geq y\}$$

