

Five applications of the “Logic for Children” project to Category Theory

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Abstract

Category Theory is usually presented in a way that is too abstract, with concrete examples of each given structure being mentioned briefly, if at all. One of the themes of the “Logic for Children” workshop, held in the UNILOG 2018, was a set of tools and techniques for drawing diagrams of categorical concepts in a canonical shape, and for drawing diagrams of particular cases of those concepts in essentially the same shape as the general case; these diagrams for a general and a particular case can be drawn side by side “in parallel” in a way that lets us transfer knowledge from the particular case to the general, and back.

In this talk we will present briefly five applications of these techniques: 1) a way to visualize planar, finite Heyting Algebras — we call them “ZHAs” — and to develop a feeling for how the logic connectives in a ZHA work; 2) a way to build a topos with a given logic (when that “logic” is a ZHA); 3) a way to represent a closure operator on a ZHA by a “slashing on that ZHA by diagonal cuts with no cuts stopping midway”; 4) a way to extend a slashing on a ZHA H to a “notion of sheafness” on the associated topos; 5) a way to start from a certain very abstract factorization of geometric morphisms between toposes, described in Peter Johnstone’s “Sketches of an Elephant” [1], and derive some intuitive meaning for what that factorization “means”: basically, we draw the diagrams, plug in it some very simple geometric morphisms, and check which ones the factorization classifies as “surjections”, “inclusions”, “closed”, and “dense”.

References

- [1] Johnstone, P. T.; *Sketches of an Elephant: A Topos Theory Compendium (Volume 1)*. Oxford University Press, 2002.

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