

8. Consider a small category \mathbf{C} . For each object B of \mathbf{C} there is a functor $D_B: \mathbf{C}/B \rightarrow \mathbf{C}$ defined by taking the domain of each arrow to B . Hence, each $T: \mathbf{C}^{\text{op}} \rightarrow \mathbf{Sets}$ yields $T_B = T \circ D^{\text{op}}: (\mathbf{C}/B)^{\text{op}} \rightarrow \mathbf{Sets}$. Define an exponential T^S by

$$T^S(B) = \text{Hom}_{(\widehat{\mathbf{C}/B})}(S_B, T_B),$$

with the evident evaluations $e_B: T^S(B) \times S(B) \rightarrow T(B)$. Show that T^S with this evaluation e is indeed the exponential in the functor category $\widehat{\mathbf{C}} = \mathbf{Sets}^{\mathbf{C}^{\text{op}}}$.