Definition. *In a category* $\mathbf{C}$ *with finite limits, a subobject classifier is a monic,* $\mathrm{true}\colon 1 \to \Omega$*, such that to every monic* $S \rightarrowtail X$ *in* $\mathbf{C}$ *there is a unique arrow* ϕ *which, with the given monic, forms a pullback square*

$$\begin{tikzcd} S \arrow[r] \arrow[d, tail] & 1 \arrow[d, tail, "\mathrm{true}"] \\ X \arrow[r, dashed, "\phi"'] & \Omega. \end{tikzcd} \tag{3}$$

In other words, every subobject is uniquely a pullback of a "universal" monic true.

This property amounts to saying that the subobject functor is representable (i.e., isomorphic to a Hom-functor). In detail, a subobject of an object X in any category $\mathbf{C}$ is an equivalence class of monics $m\colon S \rightarrowtail X$ to X (cf. the preliminaries). By a familiar abuse of language, we say that the subobject *is* S or *is* m, meaning always the equivalence class of m. Then, $\mathrm{Sub}_{\mathbf{C}}\,X$ is the set of all subobjects of X in the category $\mathbf{C}$; this set is partially ordered under inclusion. The category $\mathbf{C}$ is said to be *well-powered* when $\mathrm{Sub}_{\mathbf{C}}\,X$ is isomorphic to a small set for all X; all of our typical categories are well-powered. Now given an arrow $f\colon Y \to X$ in $\mathbf{C}$, the pullback of any monic $m\colon S \rightarrowtail X$ along f is a monic $m' : S' \rightarrowtail Y$, and the assignment $m \mapsto m'$ defines a function $\mathrm{Sub}_{\mathbf{C}}\,f\colon \mathrm{Sub}_{\mathbf{C}}\,X \to \mathrm{Sub}_{\mathbf{C}}\,Y$; when $\mathbf{C}$ is well-powered, this makes $\mathrm{Sub}_{\mathbf{C}}\colon \mathbf{C}^{\mathrm{op}} \to \mathbf{Sets}$ a functor to **Sets**. Briefly, Sub is a functor "by pullback".