Each object C of $\mathbf{C}$ gives rise to a presheaf $\mathbf{y}(C)$ on $\mathbf{C}$, defined on an object D of $\mathbf{C}$ by

$$\mathbf{y}(C)(D) = \mathrm{Hom}_{\mathbf{C}}(D, C) \tag{3}$$

and on a morphism $D' \xrightarrow{\alpha} D$, for $u\colon D \to C$, by

$$\begin{aligned} \mathbf{y}(C)(\alpha)\colon \mathrm{Hom}_{\mathbf{C}}(D, C) &\to \mathrm{Hom}_{\mathbf{C}}(D', C) \\ \mathbf{y}(C)(\alpha)(u) &= u \circ \alpha; \end{aligned} \tag{4}$$

or briefly, $\mathbf{y}(C) = \mathrm{Hom}_{\mathbf{C}}(-, C)$ is the contravariant Hom-functor. Presheaves which, up to isomorphism, are of this form are called *representable presheaves* or *representable functors*. If $f\colon C_1 \to C_2$ is a morphism in $\mathbf{C}$, there is a natural transformation $\mathbf{y}(C_1) \to \mathbf{y}(C_2)$ obtained by composition with f. This makes $\mathbf{y}$ into a functor

$$\mathbf{y}\colon \mathbf{C} \to \mathbf{Sets}^{\mathbf{C}^{\mathrm{op}}}, \qquad C \mapsto \mathrm{Hom}_{\mathbf{C}}(-, C) \tag{5}$$

from $\mathbf{C}$ to the *contravariant* functors on $\mathbf{C}$ (hence the exponent $\mathbf{C}^{\mathrm{op}}$). It is called the *Yoneda embedding.* The Yoneda embedding is a full and faithful functor. This fact is a special case of the so-called *Yoneda lemma*, which asserts for an arbitrary presheaf P on C that there is a bijective correspondence between natural transformations $\mathbf{y}(C) \to P$ and elements of the set $P(C)$:

$$\theta\colon \mathrm{Hom}_{\widehat{\mathbf{C}}}(\mathbf{y}(C), P) \xrightarrow{\sim} P(C), \tag{6}$$

defined for $\alpha\colon \mathbf{y}(C) \to P$ by $\theta(\alpha) = \alpha_C(1_C)$ (see [**CWM**, p. 61]).