

simple1 and

simple2:

$$\frac{\frac{P \wedge Q}{P} \quad \frac{\frac{P \wedge Q}{Q} \quad Q \rightarrow R}{R}}{P \wedge R}$$

$$\begin{aligned} \langle\langle P \rangle\rangle &:= \pi \langle\langle P \wedge Q \rangle\rangle \\ \langle\langle Q \rangle\rangle &:= \pi' \langle\langle P \wedge Q \rangle\rangle \\ \langle\langle R \rangle\rangle &:= \langle\langle Q \rightarrow R \rangle\rangle \langle\langle Q \rangle\rangle \\ \langle\langle P \wedge R \rangle\rangle &:= (\langle\langle P \rangle\rangle, \langle\langle R \rangle\rangle) \end{aligned}$$

$$\frac{\frac{[P \wedge Q]^1}{P} \quad \frac{\frac{[P \wedge Q]^1}{Q} \quad Q \rightarrow R}{R}}{P \wedge R} \quad \frac{P \wedge R}{P \wedge Q \rightarrow P \wedge R} \quad 1$$

$$\begin{aligned} \langle\langle P \rangle\rangle &:= \pi \langle\langle P \wedge Q \rangle\rangle \\ \langle\langle Q \rangle\rangle &:= \pi' \langle\langle P \wedge Q \rangle\rangle \\ \langle\langle R \rangle\rangle &:= \langle\langle Q \rightarrow R \rangle\rangle \langle\langle Q \rangle\rangle \\ \langle\langle P \wedge R \rangle\rangle &:= (\langle\langle P \rangle\rangle, \langle\langle R \rangle\rangle) \\ \langle\langle P \wedge Q \rightarrow P \wedge R \rangle\rangle &:= \lambda \langle\langle P \wedge Q \rangle\rangle. \langle\langle P \wedge R \rangle\rangle \end{aligned}$$

distributivity1:

$$\frac{(P \wedge R) \vee (Q \wedge R) \quad \frac{\frac{[P \wedge R]^1}{P}}{P \vee Q} \quad \frac{[P \wedge R]^1}{R}}{(P \vee Q) \wedge R} \quad \frac{\frac{[Q \wedge R]^1}{Q}}{P \vee Q} \quad \frac{[Q \wedge R]^1}{R}}{(P \vee Q) \wedge R}}{(P \vee Q) \wedge R}$$

$$\frac{\frac{(P \wedge R) \vee (Q \wedge R)}{\frac{[P \wedge R]^1}{P} \quad \frac{[Q \wedge R]^1}{Q}} \quad \frac{(P \wedge R) \vee (Q \wedge R)}{\frac{[P \wedge R]^1}{R} \quad \frac{[Q \wedge R]^1}{R}}}{\frac{(P \vee Q)}{(P \vee Q) \wedge R}} R$$

This one, which seems to require ‘ \rightarrow ’, fails in some way:

$$\frac{\frac{(P \vee Q) \wedge R}{\frac{[P]^1}{P \wedge R} \quad \frac{[Q]^1}{Q \wedge R}} \quad \frac{(P \vee Q) \wedge R}{(P \wedge R) \vee (Q \wedge R)}}{\frac{P \vee Q}{(P \wedge R) \vee (Q \wedge R)}} 1$$