Cálculo 2 - 2025.1

Aulas 43 e 44: EDOLCCs

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http://anggtwu.net/2025.1-C2.html

Links

StewPtCap17p6 (p.1020) Equações diferenciais de 2ª ordem
StewPtCap17p20 (p.1034) Caso 3: subamortecimento
StewPtApendiceHp5 (p.A51) Apêndice H: Números complexos

Leit3p22 (p.158) $D_x[c \cdot f(x)] = c \cdot D_x f(x)$
Leit3p22 (p.158) $D_x[f(x) + g(x)] = D_x f(x) + D_x g(x)$

BoyceDip3p5 (p.105) Capítulo 3: Equações lineares de 2ª ordem
BoyceDip3p11 (p.111) Seção 3.2: o operador diferencial L
BoyceDip3p13 (p.113) Teorema 3.2.2: o princípio da superposição
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ZillCullenCap4p33 (p.173) 4.3. Equações lineares homogêneas com coeficientes constantes
ZillCullenCap4p60 (p.196) Exemplo 1: ...pode ser fatorado... $(D + 3)(D + 2)$
ZillCullenCap4p61 (p.197) Exemplo 3
ZillCullenCap4p64 (p.200) Exercícios
ZillCullenEngCap4p23 (p.133) 4.3 Homogeneous Linear Equations with Constant Coefficients
ZillCullenEngCap4p40 (p.150) Factoring operators ... $(D + 3)(D + 2)$

Provas:
2jT273 2042.2, P2
2iT227 2041.2, P2
2hT279 2023.2, P2
2gT135 2023.1, P2
2fT124 2022.2, P2

Quadros:
2jQ103 2024.2
2iQ74 2024.1
2hQ61 2023.2
2gQ46 2023.1
2gQ50 2023.1

$$\begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&0\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\0&0\end{pmatrix}$$
$$\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix} = \begin{pmatrix}0&0\\0&1\end{pmatrix}$$

$$\begin{pmatrix}a&b\end{pmatrix}\begin{pmatrix}c\\d\end{pmatrix} = \begin{pmatrix}ac+bd\end{pmatrix}$$
$$\begin{pmatrix}c\\d\end{pmatrix}\begin{pmatrix}a&b\end{pmatrix} = \begin{pmatrix}ac&bc\\ad&bd\end{pmatrix}$$

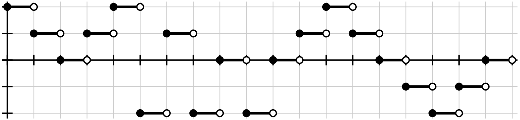
$$S = \begin{pmatrix}0&1&&&\\&0&1&&\\&&0&1&\\&&&0&1\\&&&&0\end{pmatrix} \quad 1 = \begin{pmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&1\end{pmatrix} \quad S-1 = \begin{pmatrix}-1&1&&&\\&-1&1&&\\&&-1&1&\\&&&-1&1\\&&&&-1\end{pmatrix}$$

$$v = \begin{pmatrix}v_1\\v_2\\v_3\\v_4\\v_5\end{pmatrix} \quad f = \begin{pmatrix}f(1)\\f(2)\\f(3)\\f(4)\\f(5)\end{pmatrix} \quad Sf = \begin{pmatrix}f(2)\\f(3)\\f(4)\\f(5)\\0\end{pmatrix} \quad (S-1)f = \begin{pmatrix}f(2)-f(1)\\f(3)-f(2)\\f(4)-f(3)\\f(5)-f(4)\\0-f(5)\end{pmatrix}$$

Obs: não usei isso aqui –
não deu tempo de LaTeXar tudo...

$g(x) \quad =$

$g'(x) \quad =$



```
(%i1) f   : exp( 3*x);
```
(%o1)
$$e^{3\,x}$$

```
(%i2) f   : exp(-3*x);
```
(%o2)
$$e^{-3\,x}$$

```
(%i3) f   : exp( 2*x);
```
(%o3)
$$e^{2\,x}$$

```
(%i4) fp  : diff(f,x);
```
(%o4)
$$2\,e^{2\,x}$$

```
(%i5) fpp : diff(f,x,2);
```
(%o5)
$$4\,e^{2\,x}$$

```
(%i6) Lf  : fpp + fp - 6*f;
```
(%o6)
$$0$$

```
(%i7)
```

```
(%i1) D (f) := diff(f,x);
```
(%o1)
$$D\left(f\right) := \mathrm{diff}\left(f, x\right)$$

```
(%i2) DD(f) := diff(f,x,2);
```
(%o2)
$$\mathrm{DD}\left(f\right) := \mathrm{diff}\left(f, x, 2\right)$$

```
(%i3) L (f) := D(D(f)) + D(f) - 6*f;
```
(%o3)
$$L\left(f\right) := D\left(D\left(f\right)\right) + D\left(f\right) + \left(-6\right)\,f$$

```
(%i4)   D(x^2);
```
(%o4)
$$2\,x$$

```
(%i5) D(D(x^2));
```
(%o5)
$$2$$

```
(%i6) L(x^2);
```
(%o6)
$$-\left(6\,x^2\right) + 2\,x + 2$$

```
(%i7) L(exp( 3*x));
```
(%o7)
$$6\,e^{3\,x}$$

```
(%i8) L(exp(-3*x));
```
(%o8)
$$0$$

```
(%i9) L(exp( 2*x));
```
(%o9)
$$0$$

```
(%i10)
```

Soluções não-básicas

$$\begin{aligned} M(\alpha v+\beta w) &= M(\alpha v)+M(\beta w) \\ &= \alpha(Mv)+\beta(Mw) \end{aligned}$$

$$\underbrace{(D-2)(D+3)}_{M}(\underbrace{42}_{\alpha}\underbrace{e^{2x}}_{v}+\underbrace{99}_{\beta}\underbrace{e^{-3x}}_{w})$$

$$\begin{aligned} &(D-2)(D+3)(42e^{2x}+99e^{-3x}) \\ &= 42(D-2)(D+3)e^{2x}+99(D-2)(D+3)e^{-3x} \\ &= \underbrace{42\,\underbrace{(D+3)\,\underbrace{\underbrace{\underbrace{(D-2)e^{2x}}_{De^{2x}-2e^{2x}}}_{2e^{2x}-2e^{2x}}}_{0}}_{0}}_{0} + \underbrace{99\,\underbrace{(D-2)\,\underbrace{\underbrace{\underbrace{(D+3)e^{-3x}}_{De^{-3x}+3e^{-3x}}}_{-3e^{-3x}+3e^{-3x}}}_{0}}_{0}}_{0} \end{aligned}$$

Raízes por chutar-e-testar

```
(%i1) poly0 : (x-2)*(x+5);
(%o1)
```

$$(x-2)\,(x+5)$$

```
(%i2) poly1 : expand(poly0);
(%o2)
```

$$x^2+3\,x-10$$

```
(%i3) b : ratcoef(poly1, x, 1);    /* coeficiente do x^1 */
(%o3)
```

$$3$$

```
(%i4) c : ratcoef(poly1, x, 0);    /* coeficiente do x^0 */
(%o4)
```

$$-10$$

```
(%i5)                 divisors(c);
(%o5)
```

$$\{1,2,5,10\}$$

```
(%i6) divs0 : listify(divisors(c));
(%o6)
```

$$[1,2,5,10]$$

```
(%i7)                 reverse(divs0);
(%o7)
```

$$[10,5,2,1]$$

```
(%i8)                -reverse(divs0);
(%o8)
```

$$[-10,-5,-2,-1]$$

```
(%i9) divs1 : append(-reverse(divs0), divs0);
(%o9)
```

$$[-10,-5,-2,-1,1,2,5,10]$$

```
(%i10) line(d1) := block([d2:c/d1], [d1,d2,d1*d2,d1+d2])$
(%i11) line(2);
(%o11)
```

$$[2,-5,-10,-3]$$

```
(%i12) lines0 : makelist(line(d1), d1, divs1)$
(%i13) lines1 : append([["d1", "d2", "d1*d2", "d1+d2"]], lines0)$
(%i14) apply('matrix, lines1);
(%o14)
```

$$\begin{pmatrix} \mathrm{d1} & \mathrm{d2} & \mathrm{d1*d2} & \mathrm{d1+d2} \\ -10 & 1 & -10 & -9 \\ -5 & 2 & -10 & -3 \\ -2 & 5 & -10 & 3 \\ -1 & 10 & -10 & 9 \\ 1 & -10 & -10 & -9 \\ 2 & -5 & -10 & -3 \\ 5 & -2 & -10 & 3 \\ 10 & -1 & -10 & 9 \end{pmatrix}$$

```
(%i15)
```