On the Shapes of Proofs in Basic Calculus

(Preliminary version)

http://anggtwu.net/math-b.html#2025-sbc

Eduardo Ochs

CWM

From CWM, p.84:

$$\begin{aligned} \varphi\theta g &= G\varepsilon_a \circ GFg \circ \eta_x \\ &= G\varepsilon_a \circ \eta_{Ga} \circ g \\ &= g \end{aligned}$$

$$\begin{aligned} \varphi\theta g &= \varphi(\varepsilon_a \circ Fg) && \text{by } def_\theta \\ &= G(\varepsilon_a \circ Fg) \circ \eta_x && \text{by } def_\varphi. \\ &= G\varepsilon_a \circ GFg \circ \eta_x && \text{by } respcomp_G \\ &= G\varepsilon_a \circ \eta_{Ga} \circ g && \text{by } sqcond_\eta \\ &= \mathrm{id}_{Ga} \circ g && \text{by } G\varepsilon \circ \eta G = 1 \\ &= g \end{aligned}$$

O aqui e agora não existe mais

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4)\\
F'(x) &= (F(x))'\\
&= ((6x^3)(7x^4))'\\
&= \tfrac{d}{dx}((6x^3)(7x^4))\\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3)\\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3\\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2\\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2)\\
&= (6x^3)\cdot 7\boxed{\tfrac{d}{dx}x^4} + (7x^4)(18x^2)\\
&\boxed{\overset{(10)}{=}} (6x^3)\cdot 7\cdot \boxed{4x^3} + (7x^4)(18x^2)\\
&= (6x^3)(28x^3) + (7x^4)(18x^2)\\
&= (6x^3)(28x^3) + 126x^6\\
&= 168x^6 + 126x^6\\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &=_1 (6x^3)(7x^4)\\
\boxed{F'(x)}_{2L} &=_2 (\boxed{F(x)}_{3L})'_{2R}\\
&=_3 (\boxed{(6x^3)(7x^4)}_{3R})'_{4L}\\
&=_4 \boxed{\tfrac{d}{dx}((6x^3)(7x^4))}_{4}\\
&=_5 \boxed{(6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\boxed{\tfrac{d}{dx}(6x^3)}_{6L}}_{5R}\\
&=_6 (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot \boxed{6\boxed{\tfrac{d}{dx}x^3}_{7L}}_{6R}\\
&\quad RPowx\\
&=_7 (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot \boxed{6\cdot \boxed{3x^2}_{7R}}_{8L}\\
&=_8 (6x^3)\boxed{\tfrac{d}{dx}(7x^4)}_{9L} + (7x^4)\boxed{(18x^2)}_{8R}\\
&=_9 (6x^3)\cdot \boxed{7\boxed{\tfrac{d}{dx}x^4}_{10L}}_{9R} + (7x^4)(18x^2)\\
&=_{10} (6x^3)\cdot \boxed{7\cdot \boxed{4x^3}_{10R}}_{11L} + (7x^4)(18x^2)\\
&=_{11} (6x^3)\boxed{(28x^3)}_{11R} + \boxed{(7x^4)(18x^2)}_{12L}\\
&=_{12} \boxed{(6x^3)(28x^3)}_{13L} + \boxed{126x^6}_{12R}\\
&=_{13} \boxed{168x^6}_{13R} + 126x^6{}_{14L}\\
&=_{14} \boxed{294x^6}_{14R}
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &\overset{(2)}{=} (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&\overset{(3)}{=} ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&\overset{(4)}{=} \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{array}{rcll}
F(x) & = & (6x^3)(7x^4) & \\
F'(x) & = & (F(x))' & \\
 & = & ((6x^3)(7x^4))' & \\
 & = & \frac{d}{dx}((6x^3)(7x^4)) & \\
 & \overset{(5)}{=} & (6x^3)\frac{d}{dx}(7x^4) + (7x^4)\frac{d}{dx}(6x^3) & RProd \\
 & = & (6x^3)\frac{d}{dx}(7x^4) + (7x^4) \cdot 6\frac{d}{dx}x^3 & \\
 & = & (6x^3)\frac{d}{dx}(7x^4) + (7x^4) \cdot 6 \cdot 3x^2 & \\
 & = & (6x^3)\frac{d}{dx}(7x^4) + (7x^4)(18x^2) & \\
 & = & (6x^3) \cdot 7\frac{d}{dx}x^4 + (7x^4)(18x^2) & \\
 & = & (6x^3) \cdot 7 \cdot 4x^3 + (7x^4)(18x^2) & \\
 & = & (6x^3)(28x^3) + (7x^4)(18x^2) & \\
 & = & (6x^3)(28x^3) + 126x^6 & \\
 & = & 168x^6 + 126x^6 & \\
 & = & 294x^6 &
\end{array}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\boxed{\tfrac{d}{dx}(6x^3)} \\
&\boxed{\overset{(6)}{=}} \quad (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot \boxed{6\tfrac{d}{dx}x^3} \quad RMC \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\,\tfrac{d}{dx}x^3 \\
&\overset{(7)}{=} (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \quad RPowx \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&\overset{(8)}{=} (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\boxed{\tfrac{d}{dx}(7x^4)} + (7x^4)(18x^2) \\
&\overset{(9)}{=} (6x^3)\cdot \boxed{7\tfrac{d}{dx}x^4} + (7x^4)(18x^2) \quad RMC \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\boxed{\tfrac{d}{dx}x^4} + (7x^4)(18x^2) \\
&\overset{(10)}{=} (6x^3)\cdot 7\cdot \boxed{4x^3} + (7x^4)(18x^2) \quad RPowx \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot \boxed{7\cdot 4x^3} + (7x^4)(18x^2) \\
&\overset{(11)}{=} (6x^3)\boxed{(28x^3)} + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&\overset{(12)}{=} (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= \boxed{(6x^3)(28x^3)} + 126x^6 \\
&\overset{(13)}{=} \boxed{168x^6} + 126x^6 \\
&= 294x^6
\end{aligned}
$$

$$
\begin{aligned}
F(x) &= (6x^3)(7x^4) \\
F'(x) &= (F(x))' \\
&= ((6x^3)(7x^4))' \\
&= \tfrac{d}{dx}((6x^3)(7x^4)) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\tfrac{d}{dx}(6x^3) \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\tfrac{d}{dx}x^3 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)\cdot 6\cdot 3x^2 \\
&= (6x^3)\tfrac{d}{dx}(7x^4) + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\tfrac{d}{dx}x^4 + (7x^4)(18x^2) \\
&= (6x^3)\cdot 7\cdot 4x^3 + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + (7x^4)(18x^2) \\
&= (6x^3)(28x^3) + 126x^6 \\
&= 168x^6 + 126x^6 \\
&\overset{(14)}{=} 294x^6
\end{aligned}
$$