

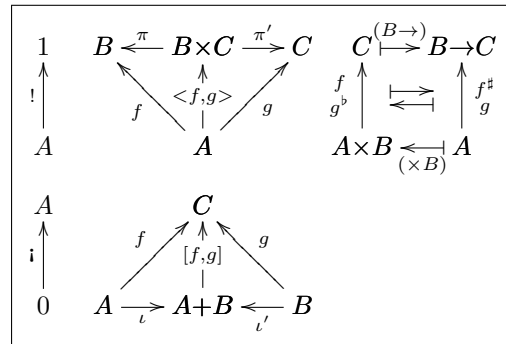
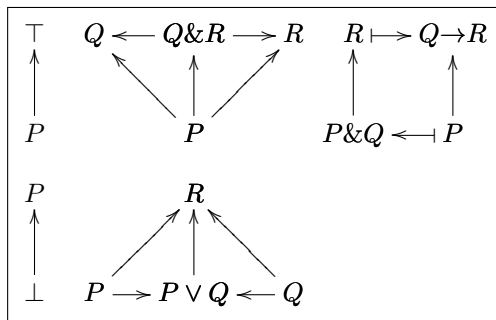
### 1. A comparison between HAs and Cartesian Closed Categories:

$$\begin{array}{l}
 (id) \quad \forall P.(P \leq P) \\
 (comp) \quad \forall P, Q, R.(P \leq Q) \& (Q \leq R) \rightarrow (P \leq R) \\
 \\
 (\top) \quad \forall P.(P \leq \top) \\
 (\perp) \quad \forall P.(\perp \leq P) \\
 (\&) \quad \forall P, Q, R.(P \leq Q \& R) \leftrightarrow (P \leq Q) \& (P \leq R) \\
 (\vee) \quad \forall P, Q, R.(P \vee Q \leq R) \leftrightarrow (P \leq R) \& (Q \leq R) \\
 (\rightarrow) \quad \forall P, Q, R.(P \leq Q \rightarrow R) \leftrightarrow (P \& Q \leq R) \\
 \\
 (\neg) \quad \neg P := P \rightarrow \perp \\
 (\leftrightarrow) \quad P \leftrightarrow Q := (P \rightarrow Q) \& (Q \rightarrow P)
 \end{array}$$

$$\begin{array}{l}
 (id) \quad id_A \in \text{Hom}(A, A) \\
 (comp) \quad (;)_{A,B,C} : \text{Hom}(A, B) \times \text{Hom}(B, C) \rightarrow \text{Hom}(A, C) \\
 \\
 (\top) \quad \forall A.(\text{Hom}(A, 1) \simeq 1) \\
 (\perp) \quad \forall A.(\text{Hom}(0, A) \simeq 1) \\
 (\&) \quad \forall A, B, C.(\text{Hom}(A, B \times C) \simeq \text{Hom}(A, B) \times \text{Hom}(A, C)) \\
 (\vee) \quad \forall A, B, C.(\text{Hom}(A + B, C) \simeq \text{Hom}(A, C) \times \text{Hom}(B, C)) \\
 (\rightarrow) \quad \forall A, B, C.(\text{Hom}(A, C^B) \simeq \text{Hom}(A \times B, C))
 \end{array}$$

$$\begin{array}{l}
 \frac{}{P \leq \top} ! \quad \frac{P \leq Q \quad P \leq R}{P \leq Q \& R} \langle, \rangle \quad \frac{P \leq Q \rightarrow R}{P \& Q \leq R} \flat \\
 \\
 \frac{}{\perp \leq P} i \quad \frac{P \leq P \vee Q \quad Q \leq P \vee Q}{P \vee Q \leq R} \flat, \flat'
 \end{array}$$

$$\begin{array}{l}
 \frac{}{! : A \rightarrow 1} i \quad \frac{f : A \rightarrow B \quad g : A \rightarrow C}{\langle f, g \rangle : A \rightarrow B \times C} \langle, \rangle \quad \frac{f : A \times B \rightarrow C}{f^\# : A \rightarrow B \rightarrow C} \sharp \\
 \\
 \frac{}{i : 0 \rightarrow A} i \quad \frac{f : A \rightarrow B \quad g : A \rightarrow C}{\langle f, g \rangle : A \rightarrow B \times C} \langle, \rangle \quad \frac{f : A \rightarrow A + B \quad g : B \rightarrow A + B}{\flat : A \rightarrow A + B} \flat, \flat'
 \end{array}$$



Logic and Categories, or: Heyting Algebras for Children

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<http://angg.twu.net/math-b.html#istanbul>

2. Calculating  $21 \& 12$  and  $21 \rightarrow 12$  by brute force:

$$\forall P. (P \leq \underbrace{Q}_{21} \& \underbrace{R}_{12}) \leftrightarrow ( \underbrace{P \leq Q}_{21} ) \& ( \underbrace{P \leq R}_{12} )$$

$$\underbrace{\lambda P. P \leq ? =}_{\lambda P. P \leq ? =}$$

$$\underbrace{\lambda P. P \leq 21 = \quad \lambda P. P \leq 12 =}_{\lambda P. P \leq 21 \& P \leq 12 =}$$

$$\begin{array}{c} 0 \\ 0 \ 0 \\ 0 \ 0 \ 0 \\ 0 \ 1 \ 1 \ 0 \\ 1 \ 1 \ 1 \\ 1 \end{array} \quad \begin{array}{c} 0 \\ 0 \ 0 \\ 0 \ 0 \ 0 \\ 0 \ 1 \ 1 \ 0 \\ 1 \ 1 \ 1 \\ 1 \end{array} \quad \begin{array}{c} 0 \\ 0 \ 0 \\ 0 \ 0 \ 0 \\ 0 \ 0 \ 1 \ 0 \\ 0 \ 1 \ 1 \ 1 \\ 1 \ 1 \end{array}$$

$$\begin{array}{c} 0 \\ 0 \ 0 \\ 0 \ 0 \ 0 \\ 0 \ 1 \ 1 \ 0 \\ 1 \ 1 \ 1 \\ 1 \end{array}$$

$$(P \leq \underbrace{Q}_{21} \rightarrow \underbrace{R}_{12}) \leftrightarrow ( \underbrace{P \& Q}_{21} \leq \underbrace{R}_{12} )$$

$$\underbrace{\lambda P. P \leq ? =}_{\lambda P. P \leq ? =}$$

$$\underbrace{\lambda P. P \& 21 =}_{\lambda P. (P \& 21) \leq 12 =}$$

$$\begin{array}{c} 0 \\ 0 \ 0 \\ 0 \ 0 \ 0 \\ 0 \ 0 \ 1 \ 1 \\ 0 \ 1 \ 1 \ 1 \\ 1 \end{array} \quad \begin{array}{c} 21 \\ 21 \ 21 \\ 21 \ 21 \ 11 \\ 20 \ 21 \ 11 \ 01 \\ 20 \ 11 \ 01 \\ 10 \ 01 \\ 00 \end{array}$$

$$\begin{array}{c} 0 \\ 0 \ 0 \\ 0 \ 0 \ 1 \\ 0 \ 1 \ 1 \ 1 \\ 1 \ 1 \ 1 \\ 1 \end{array}$$