

$$\begin{array}{ccccc}
 \mathcal{A}W & \xrightarrow{\text{Id}_{\mathcal{A}W}} & \mathcal{A}W & \xrightarrow{\alpha_W} & \mathcal{B}W \\
 \downarrow & & \downarrow \mathcal{A}(f) & & \downarrow \mathcal{B}(f) \\
 & & \Leftrightarrow & & \\
 & & a_f & & \\
 & & \downarrow & & \downarrow \\
 \mathcal{A}(gf) & \Leftrightarrow & \mathcal{A}X & \xrightarrow{\alpha_X} & \mathcal{B}X \\
 & & \downarrow & & \downarrow \\
 & & \mathcal{A}(g) & \Leftrightarrow & \mathcal{B}(g) \\
 \downarrow & & \downarrow & & \downarrow \\
 \mathcal{A}Y & \xrightarrow{\text{Id}_{\mathcal{A}Y}} & \mathcal{A}Y & \xrightarrow{\alpha_Y} & \mathcal{B}Y
 \end{array}
 = \quad
 \begin{array}{ccccc}
 \mathcal{A}W & \xrightarrow{\alpha_W} & \mathcal{B}W & \xrightarrow{\text{Id}_{\mathcal{B}W}} & \mathcal{B}W \\
 \downarrow & & \downarrow \mathcal{B}(gf) & & \downarrow \mathcal{B}(g)\mathcal{B}(f) \\
 & & \Leftrightarrow & & \\
 & & \alpha_{gf} & & b_{gf} \\
 & & \downarrow & & \downarrow \\
 \mathcal{A}(gf) & \Leftrightarrow & \mathcal{B}(gf) & \Leftrightarrow & \mathcal{B}(g)\mathcal{B}(f) \\
 \downarrow & & \downarrow & & \downarrow \\
 \mathcal{A}Y & \xrightarrow{\alpha_Y} & \mathcal{B}Y & \xrightarrow{\text{Id}_{\mathcal{B}Y}} & \mathcal{B}Y
 \end{array}$$

