Visualization as Restructuring and thus a Source of Logical Paradox

We survey and systematize the ways our minds organize and visualize thoughts. We then observe their relevance in explaining different kinds of logical paradox. We also show where they arise in math.

We were inspired by educator Kestas Augutis's vision that every high school student write three books (a chronicle, a thesaurus, and an encyclopedia) so as to master three kinds of thinking (sequential, hierarchical, and network). We thus collected dozens of examples of how we organize our thoughts. Surprisingly, we never use sequences, hierarchies or networks in isolation. Instead, we use them in pairs:

- Evolution: A hierarchy (of variations) is restructured with a sequence (of times).
- Atlas: A network (of adjacency relations) is restructured with a hierarchy (of global and local views).
- Handbook: A sequence (of instructions) is restructured with a network (of loops and branches).
- Chronicle: A sequence (of events in time) is restructured with a hierarchy (of time periods).
- Catalog: A hierarchy (of concepts) is restructured with a network (of cross-links).
- Odyssey: A network (of states) is restructured with a sequence (of steps).

In general, a first, large, comprehensive structure grows so robust that we restructure it with a second, smaller, different structure of multiple vantage points.

In a separate investigation, we listed and grouped paradoxes. This yielded the following six themes:

- Concepts may be inexact. (The paradox of an evolution.) We can't specify exactly at what point in the womb a child becomes conscious, or at what point in evolution two species diverge.
- The whole is not the sum of the parts. (The paradox of an atlas.) If we replace all of the parts of an automobile, and then build a copy with all of the old parts, which is the original?
- Our attention affects what we observe. (The paradox of a handbook.) Achilles can never catch a tortoise if we keep measuring the distance between them.
- There may be a limited contradiction. (The paradox of a chronicle.) How can we reliably learn from one who has ever made a mistake?
- We cannot make explicit all relevant assumptions. (The paradox of a catalog.) 10+4 may equal 2 if we happen to be thinking about a clock.
- We can choose differently in the same circumstances. (The paradox of an odyssey.) "I am lying when I say 'I am lying.'"

Each type of paradox brings to light the fundamental gap between the (seemingly infinite) primary comprehensive structure and the (manifestly finite) secondary structure which organizes our vantage points. Our mind visualizes a qualitative but illusory relationship between the two structures.

These same six restructurings arose in a broader investigation which yielded 24 ways of figuring things out in mathematics. We identify the six restructurings with six axioms of set theory: Pairing, Extensionality, Well-ordering, Power set, Union and Regularity.