Category theory and its foundations: the role of diagrams and other "intuitive" material

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When analyzing, in *Tool and object* [1], the historical development of category theory and the early debate on its foundations, I was led to discuss some general philosophical aspects of the formation of new mathematical concepts (in learners and in a community as a whole) and of mathematical research programmes; motivating examples were discussed under the headings of "intended models" and "technical common sense". It turned out to be crucial to focus on the respective background of the people involved in these processes, in particular, the attitude of "people without expertise in a certain area" was shown to play a role.

This observation lends itself to discussion within the perspective of the workshop (which speaks about such groups of people as "children in a wider sense of the term"); therefore, the talk will review this issue to some extent. A special focus will be laid on the role of diagrams in the debates on category theory. On the one hand, I intend to compare the role of diagrams played in proofs of category theory with the role of diagrams played in proofs of classical Euclidean geometry (as analyzed by Manders [2], among others). In both cases, one should focus on the ways in which a diagram is used to *prove* a proposition, on the one hand, or to *display* a proposition, on the other. And there is a tension playing an eminent role, in my opinion, in the foundational debate, namely the tension between diagrams as displaying propositions about finite sets of objects of a category on the one hand and the consideration of a category as an infinite diagram (or graph) on the other.

References

1. R. Krömer, *Tool and object. A history and philosophy of category theory.* Birkhäuser, 2007
2. K. Manders, "Diagram-Based Geometric Practice"/"The Euclidean Diagram (1995)". Chapters 3 and 4 in P. Mancosu, ed., *The Philosophy of Mathematical Practice.* Oxford Univ Pr, 2008, pp. 6579, 80133.